BH HORIZONS

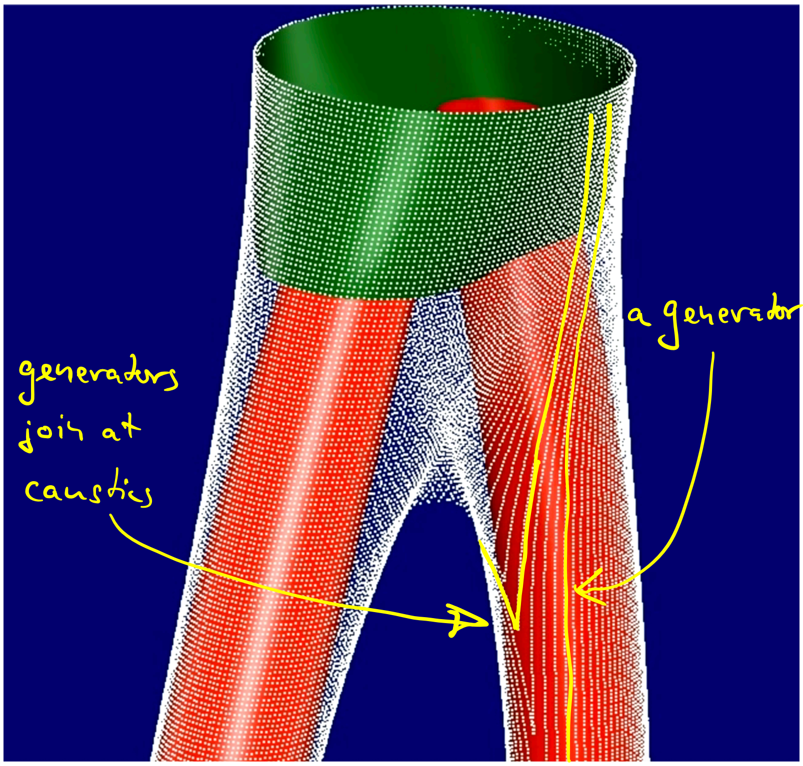
- Event horizons (global. Require 4-D data)
- Apparent horizons (local in time: Require 3-D data at $t = \text{const}$)

Event-Horizons

- foliated by null geodesics ("generators" of Horizon)
- which geodesics determined at late time
  - informally: those that don't fall into BH and neither escape
  - formally: boundary of past domain of dependence of $\mathcal{J}^+$
- new geodesics can enter horizon at caustics
- A_{EH} grows when geodesics diverge

Merger of two BHs "pair of pants"

space-time diagram



Key concept: expansion of null-geodesic congruence.

$$\Theta = \nabla_\mu k^\mu$$

Raychaudhuri's eqn for null congruences

$$\mathcal{L}_k \Theta = -\frac{1}{2}\Theta^2 - \sigma^{\mu\nu}\sigma_{\mu\nu} + \omega^{\mu\nu}\omega_{\mu\nu} - \underbrace{R_{\mu\nu}k^\mu k^\nu}$$

$= 0$ in vacuum
≥ 0 for matter satisfying weak energy condition

shear of congruence
(= energy flux across)

twist of congruence
$= 0$ b/c surface forming

$$\Rightarrow \frac{d\Theta}{d\lambda} \leq -\frac{1}{2}\Theta^2$$

affine param along geodesic

$\Theta < 0 \Rightarrow \Theta \to -\infty$ in finite affine parameter

$\Rightarrow$ Penrose singularity thm $\Rightarrow$ not on EH

$\Rightarrow \Theta \geq 0$ on EH

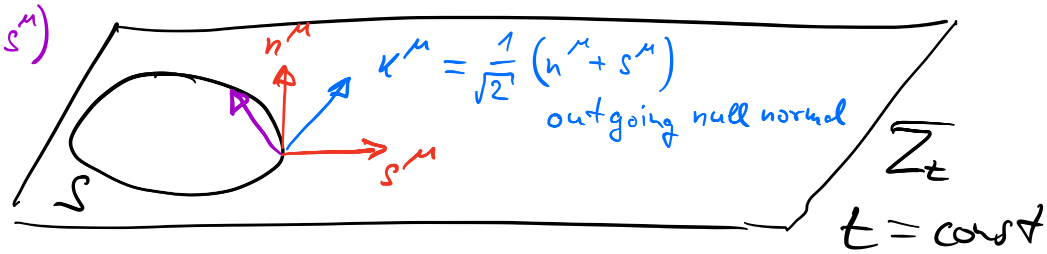
$\Theta = 0$ on EH w/ constant area (non-growing)

Apparent Horizon

outermost marginally outer trapped surface MOTS

n^μ: unit normal $\perp$ to Σ_t $\quad n^\mu n_\mu = -1$

$\ell^\mu = \frac{1}{\sqrt{2}}(n^\mu - s^\mu)$
ingoing null normal



s^μ: unit normal to S
within Σ_t

$s^\mu s_\mu = +1$

null:

$$k_\mu k^\mu = \frac{1}{2}(n+s)(n+s)$$
$$= \frac{1}{2}(\underbrace{n^2}_{-1} + 2\underbrace{n\cdot s}_{0} + \underbrace{s^2}_{+1})$$
$$= 0$$

MOTS: $\quad \Theta_{(k)} = \nabla_\mu k^\mu = 0$

In 3+1:

induced metric on Horizon:

$$m_{\mu\nu} = g_{\mu\nu} + k_\mu \ell_\nu + k_\nu \ell_\mu$$

$$\Theta_{(k)} = m^{\mu\nu} \nabla_\mu k_\nu$$
$$= \frac{1}{\sqrt{2}} m^{\mu\nu}(\nabla_\mu n_\nu + \nabla_\mu s_\nu)$$
$$= \frac{1}{\sqrt{2}} m^{\mu\nu}(-K_{\mu\nu} + D_\mu s_\nu) \qquad (*)$$

all tensors in (*) are spatial, so can switch to spatial indices.

Moreover $m_{ij} = \gamma_{ij} - s_i s_j$

$$\Longrightarrow \quad \Theta_{(k)} = \frac{1}{\sqrt{2}} \left(D_i s^i - K + K_{ij} s^i s^j \right) \qquad (**)$$

$\therefore$ to find AH (or more generally any MOTS), find topologically spherical surface S within Σ_t that satisfies $\Theta_{(k)} = 0$ everywhere on S.

AH (MOTS) depend <u>only</u> on data on Σ_t (γ_{ij}, K_{ij}).

For stationary BHs (where EH has $\Theta = 0$), AH = EH

For growing BHs (where EH has $\Theta > 0$), $\underset{\Theta=0}{\text{AH}}$ inside $\underset{\Theta>0}{\text{EH}}$

Common AH appears discontinuously

AHs of individual BHs