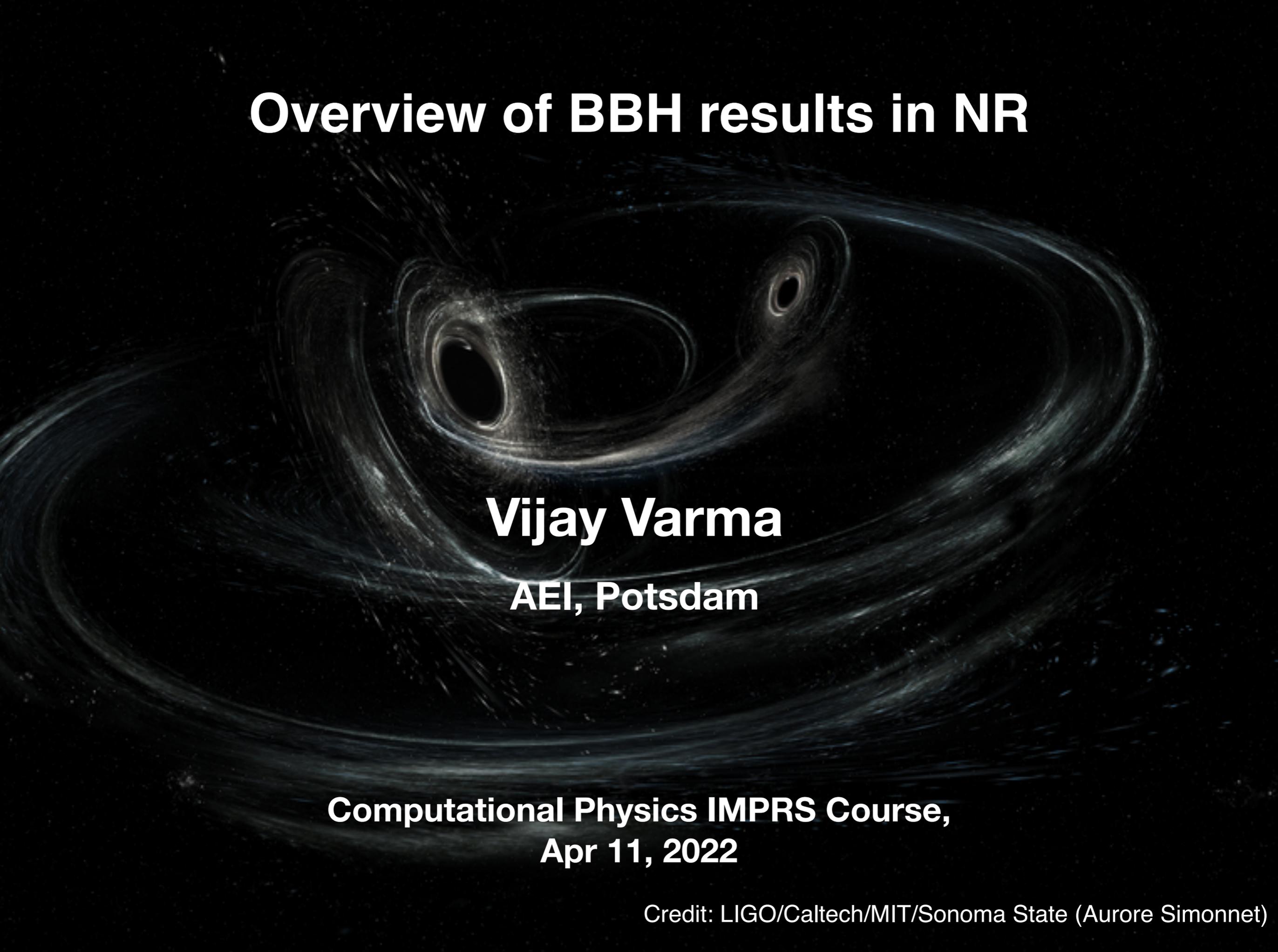


# Overview of BBH results in NR



**Vijay Varma**

**AEI, Potsdam**

**Computational Physics IMPRS Course,  
Apr 11, 2022**

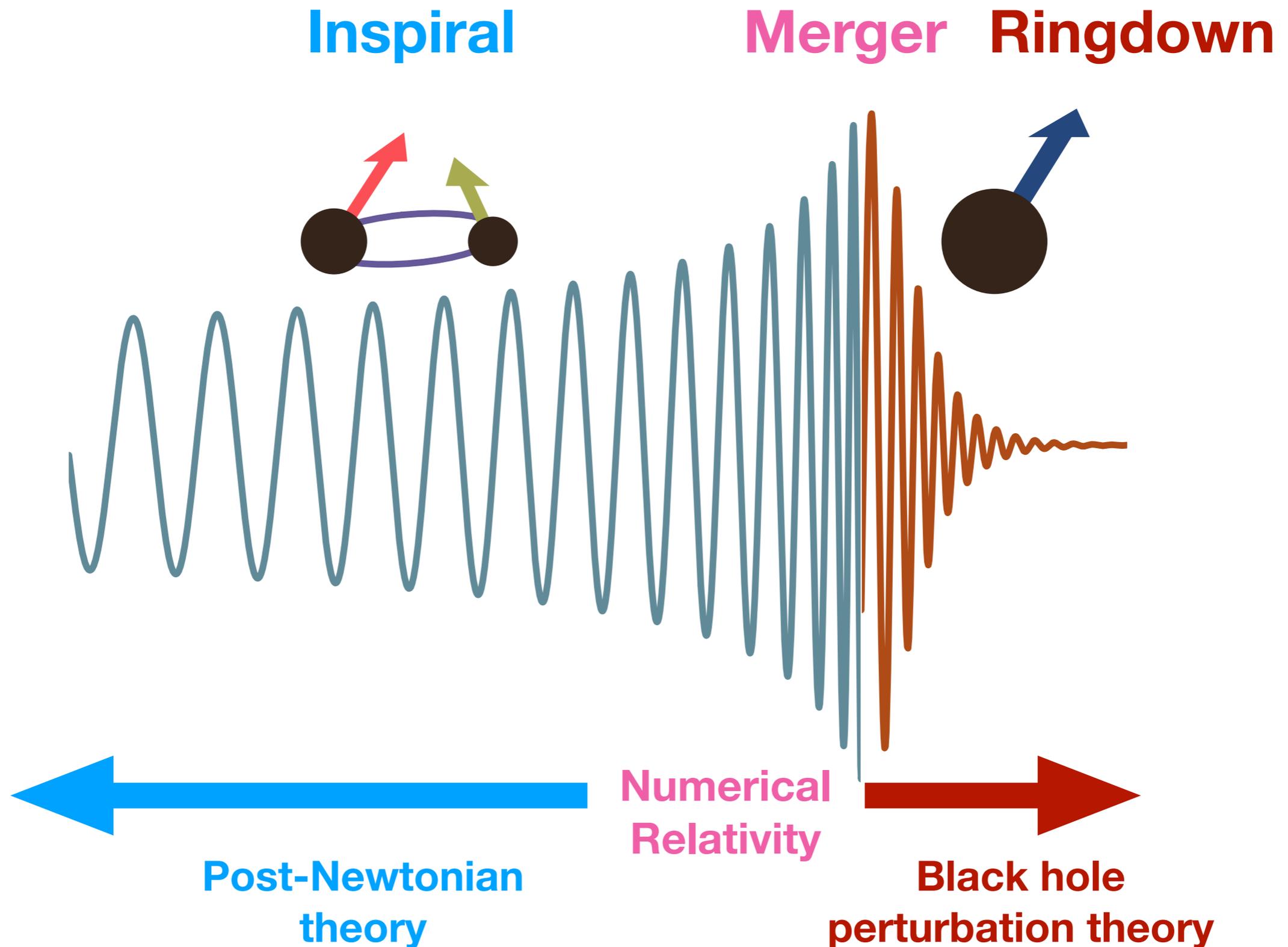
Credit: LIGO/Caltech/MIT/Sonoma State (Aurore Simonnet)

# Outline

- Brief history of NR
- Important BBH phenomena everyone should know:
  - Higher order modes
  - Junk radiation.
  - Precession
  - “Hang-up” effect
  - Eccentricity
  - Kicks
- Status of the field and frontiers

# Part I: Brief history of NR

# Stages of a binary black hole



# The very beginning

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ANNALS OF PHYSICS: 29, 304-331 (1964)

## The Two-Body Problem in Geometrodynamics

SUSAN G. HAHN

*International Business Machines Corporation, New York, New York*

AND

RICHARD W. LINDQUIST

The numerical calculations were carried out on an IBM 7090 electronic computer. The parameters  $a$  and  $\mu_0$  were both set equal to unity; the mesh lengths were assigned the values  $h_1 = 0.02$ ,  $h_2 = \pi/150 \approx 0.021$ , yielding a 51  $\times$  151 mesh. The calculations of all unknown functions, including a great number of input-output operations and some built-in checking procedures, took approximately four minutes per time step. Different check routines indicated that results close to the point  $\mu = 0$ ,  $\eta = 0$  lost accuracy fairly quickly. Since these would, in the long run, influence meshpoints further away, the computations were stopped after the 50th time step, when the total time elapsed was approximately 1.8. Some of the results are shown in Table I.

Evolution  
time 1.8M

# 40 years later...

VOLUME 92, NUMBER 21

PHYSICAL REVIEW LETTERS

week ending  
28 MAY 2004

## Numerical Simulation of Orbiting Black Holes

Bernd Brügmann, Wolfgang Tichy, and Nina Jansen

*Center for Gravitational Physics and Geometry and Center for Gravitational Wave Physics, Penn State University,  
University Park, Pennsylvania 16802, USA*

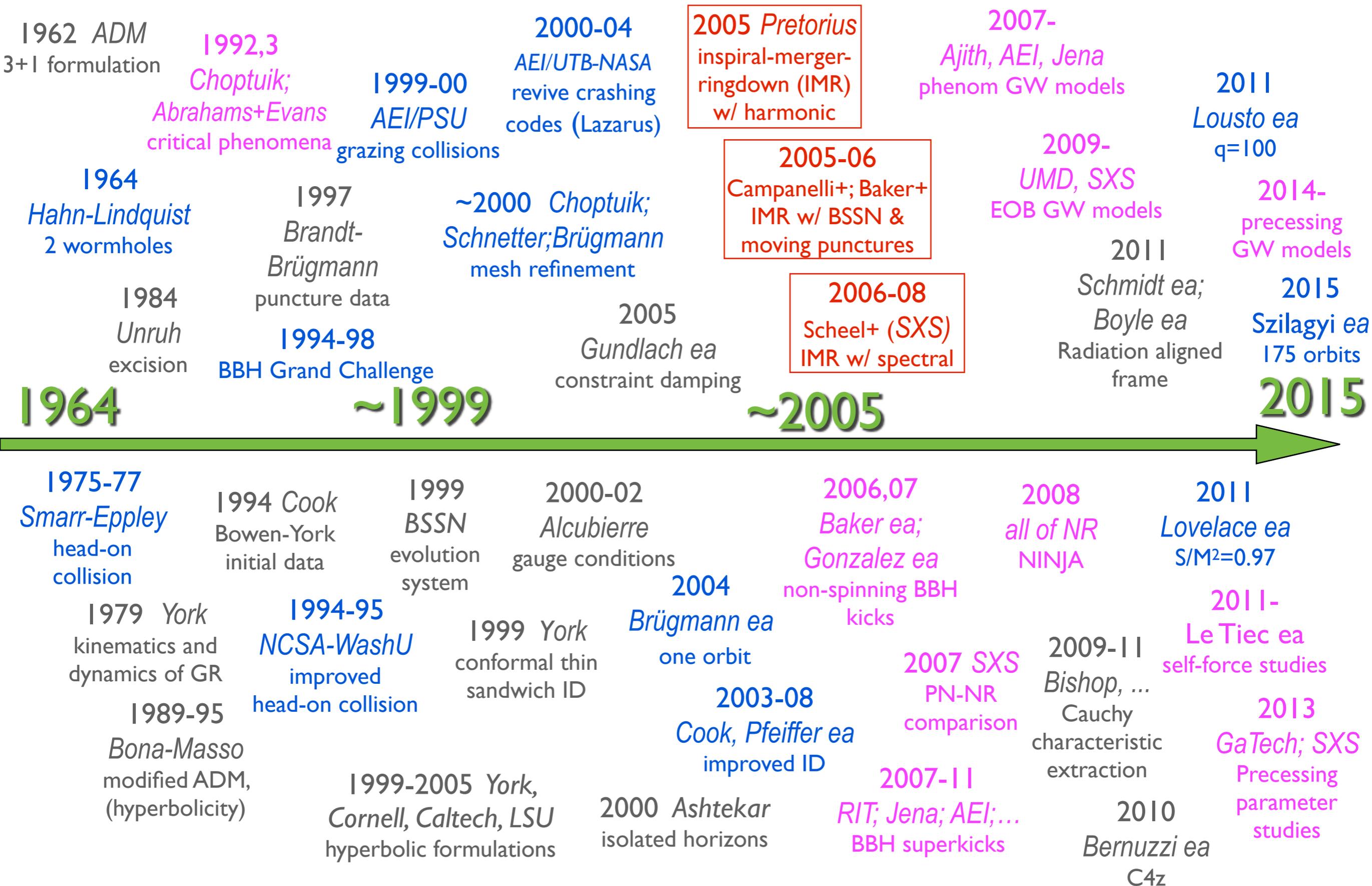
(Received 26 December 2003; published 24 May 2004)

We present numerical simulations of binary black hole systems which **for the first time last for about one orbital period** for close but still separate black holes as indicated by the absence of a common apparent horizon. An important part of the method is the construction of comoving coordinates, in which both the angular and the radial motion are minimized through a dynamically adjusted shift condition. We use fixed mesh refinement for computational efficiency.

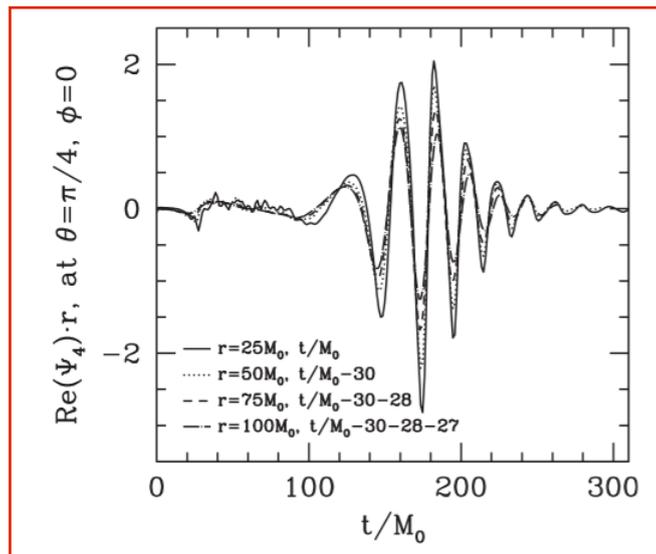
DOI: 10.1103/PhysRevLett.92.211101

PACS numbers: 04.25.Dm, 04.30.Db, 95.30.Sf

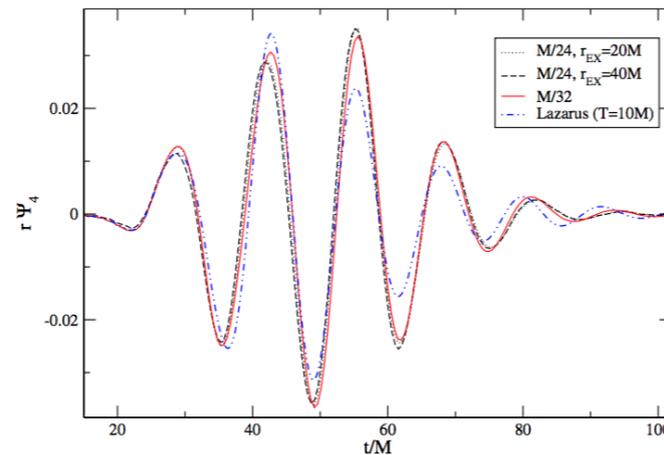
# First 50 years of numerical relativity for BBH



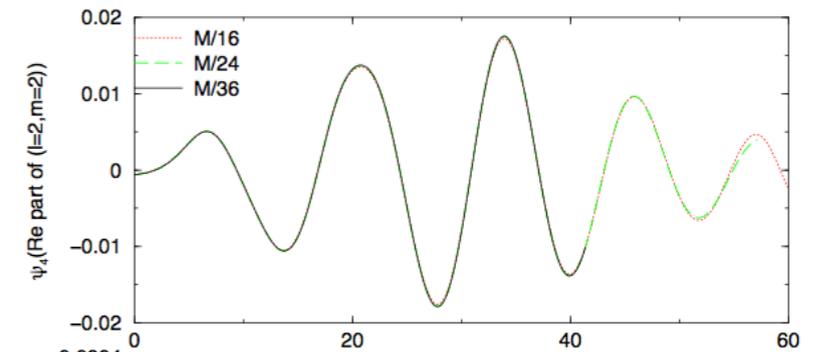
# First BBH simulations



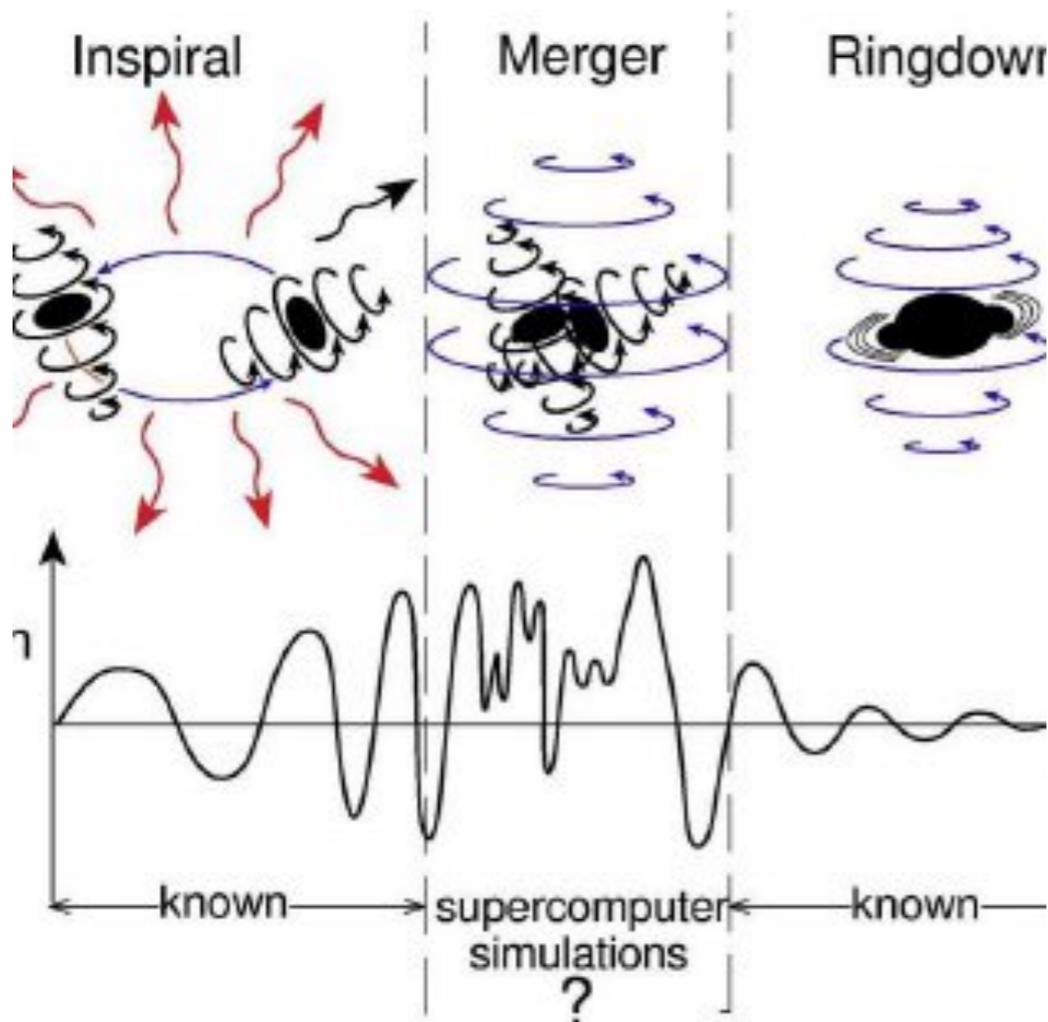
*Pretorius 05*



*Baker+06*



*Campanelli+06*



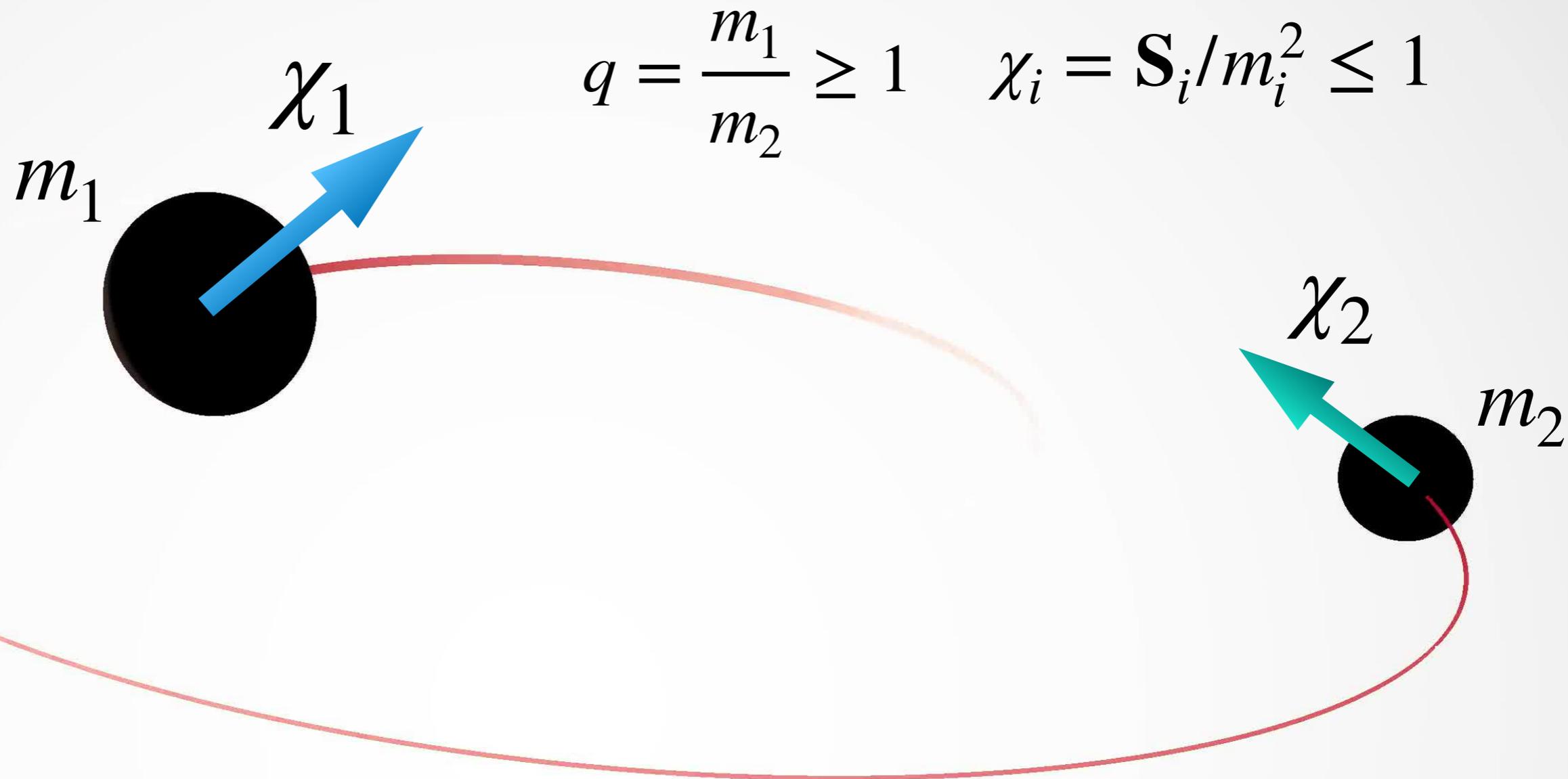
*Kip Thorne*

Important early result:  
**Simplicity of merger.**

Continuous transition  
inspiral → ringdown

# Part II: Important BBH phenomena

# Binary black hole parameters



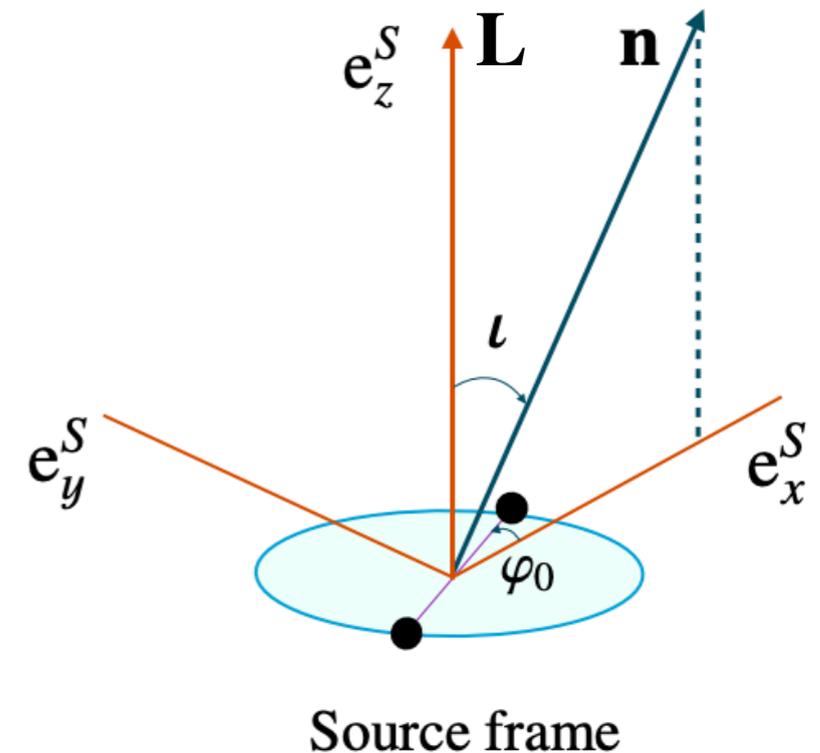
$$\Lambda = \{q, \chi_{1x}, \chi_{1y}, \chi_{1z}, \chi_{2x}, \chi_{2y}, \chi_{2z}\}$$

# Higher order modes

$$h = h_+ - i h_\times$$

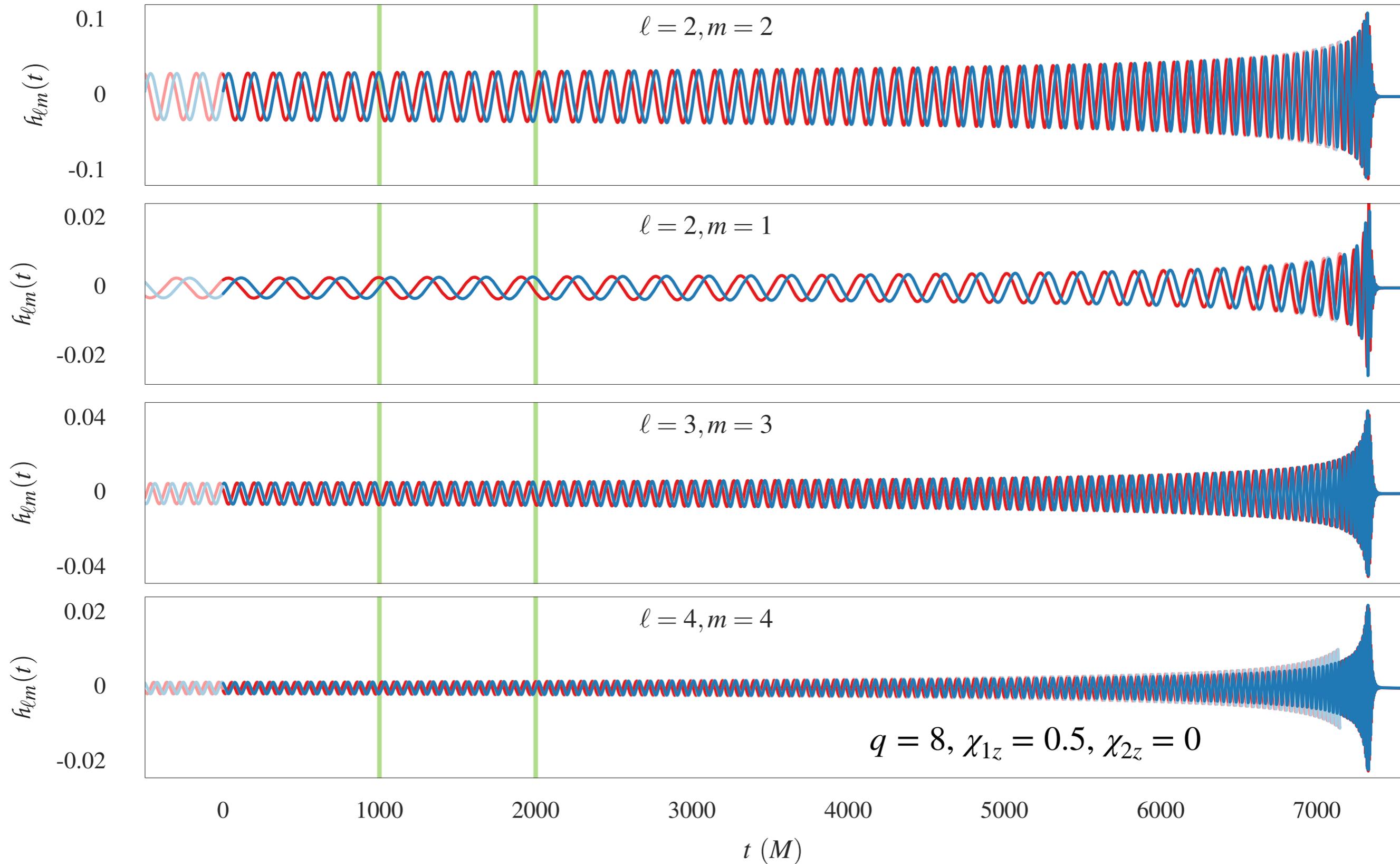
$$h(\iota, \varphi_0, t) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} -2Y_{\ell m}(\iota, \varphi_0) h_{\ell m}(t)$$

$$h_{\ell m} = A_{\ell m} e^{i\phi_{\ell m}} ; \quad \omega_{lm} = \frac{d\phi_{\ell m}}{dt} \sim m \omega_{\text{orb}}$$

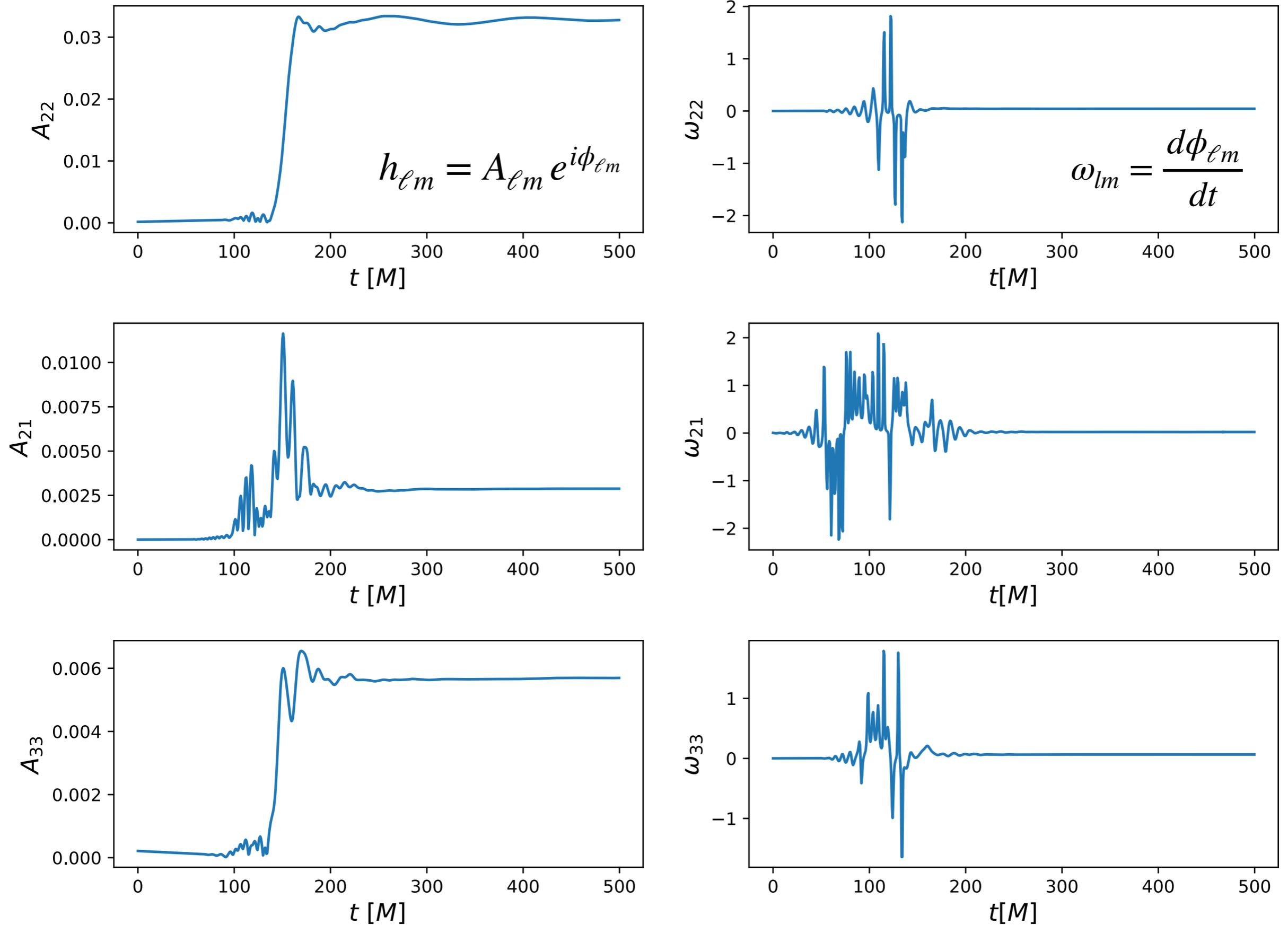


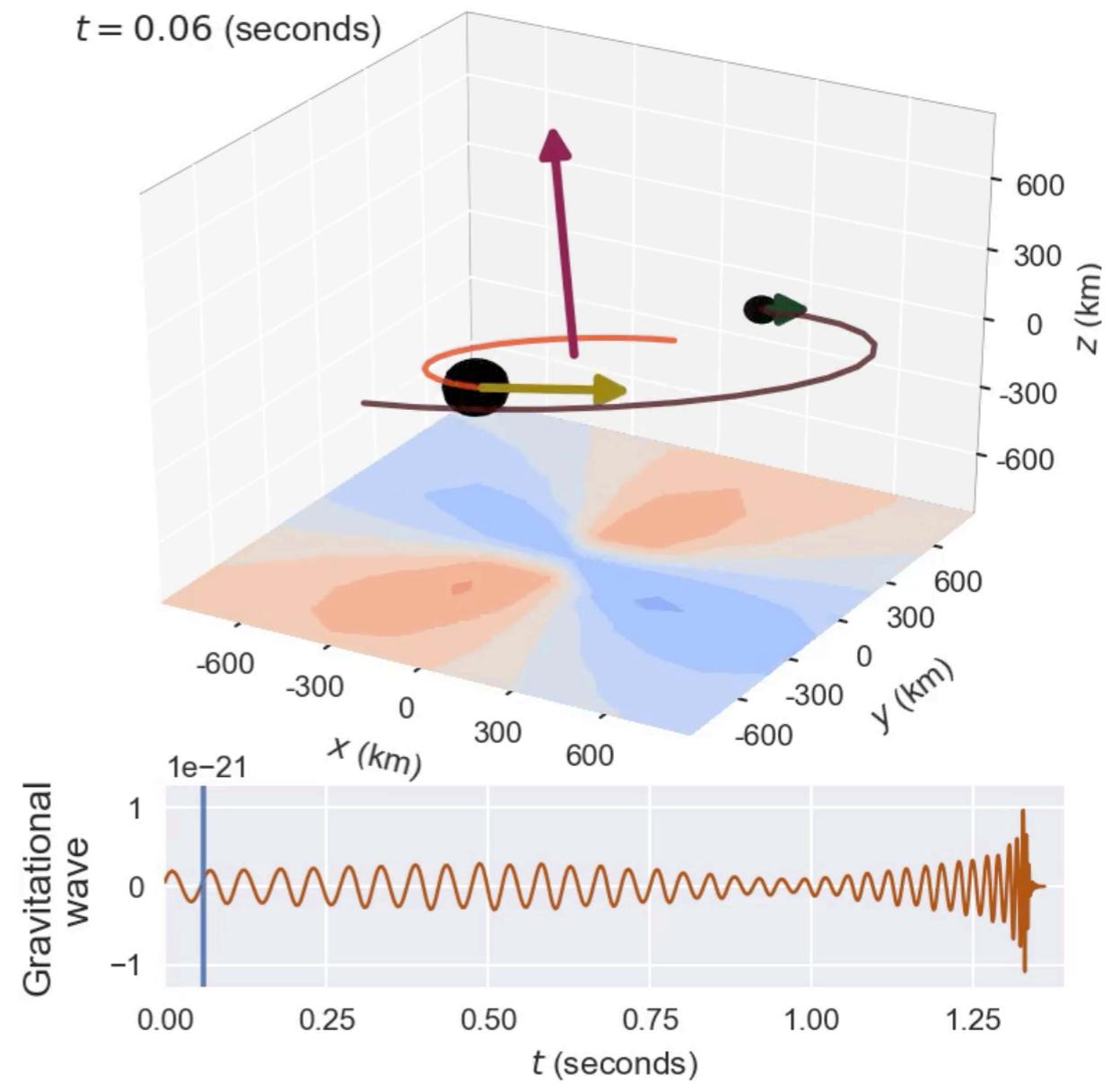
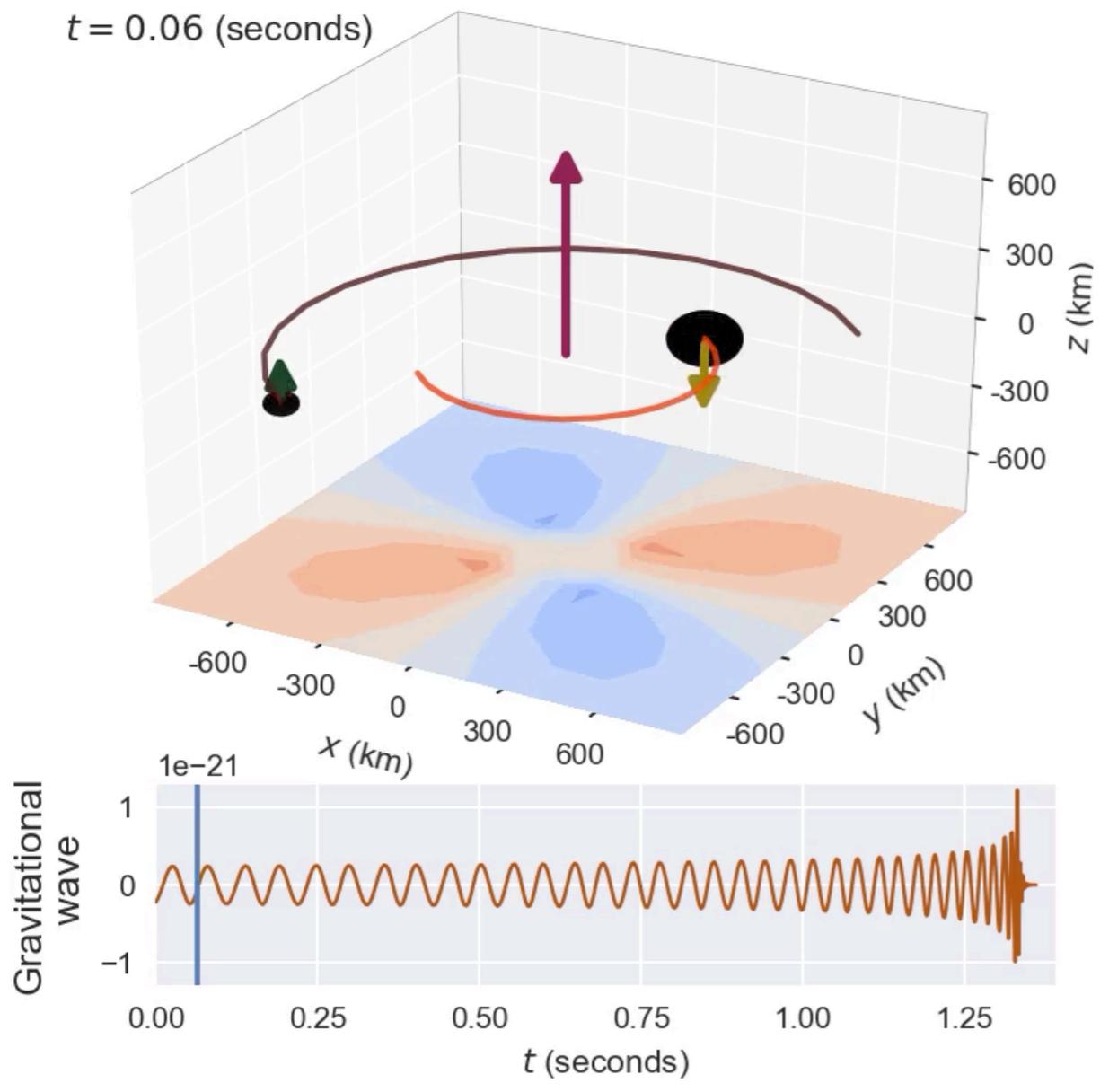
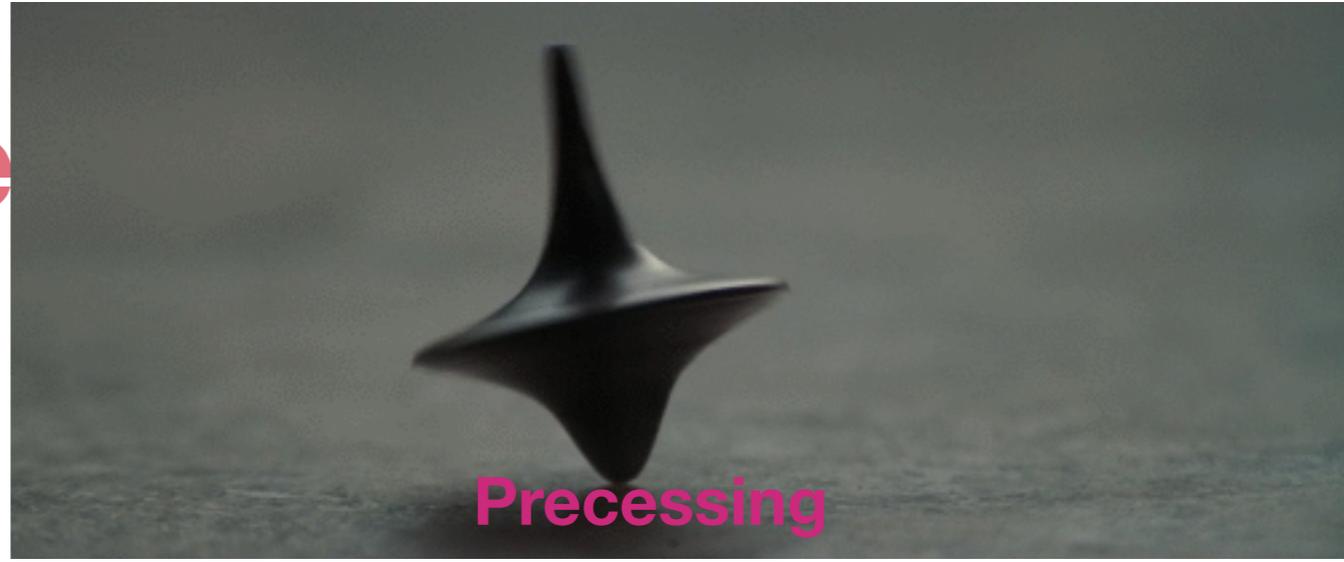
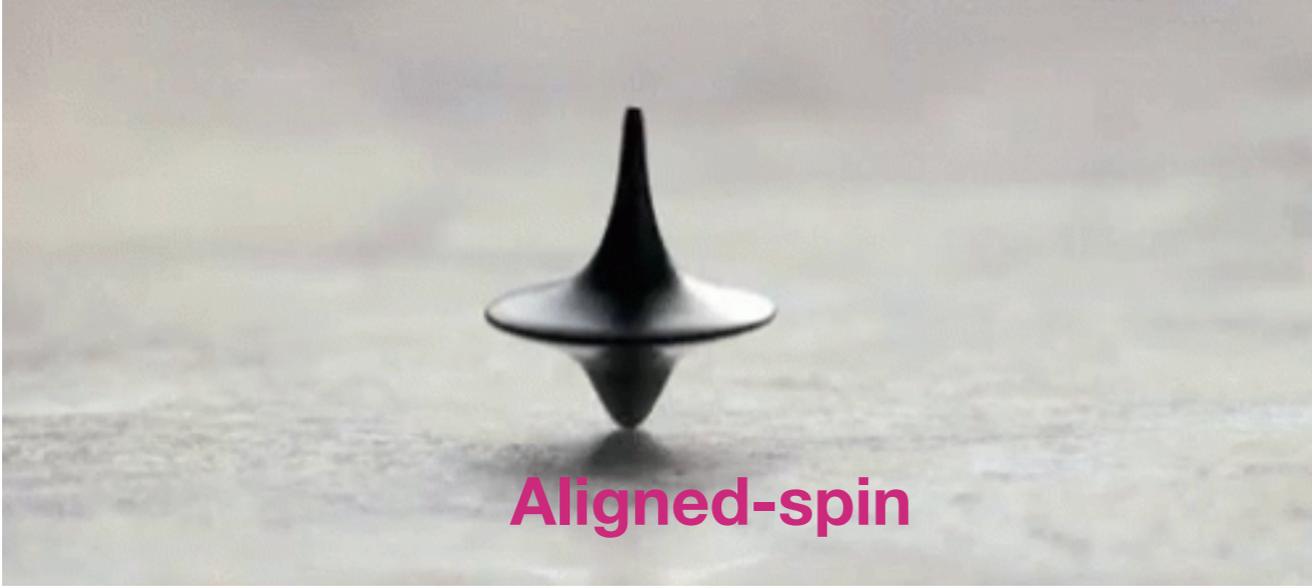
- **Quadrupole modes,  $(2, \pm 2)$ , are typically dominant.**
- **Other modes are referred to as higher (order) modes, higher harmonics, nonquadrupole modes, or subdominant modes.**
- **Become important at large  $q$  or large inclination angles  $\theta$ .**

# Higher order modes

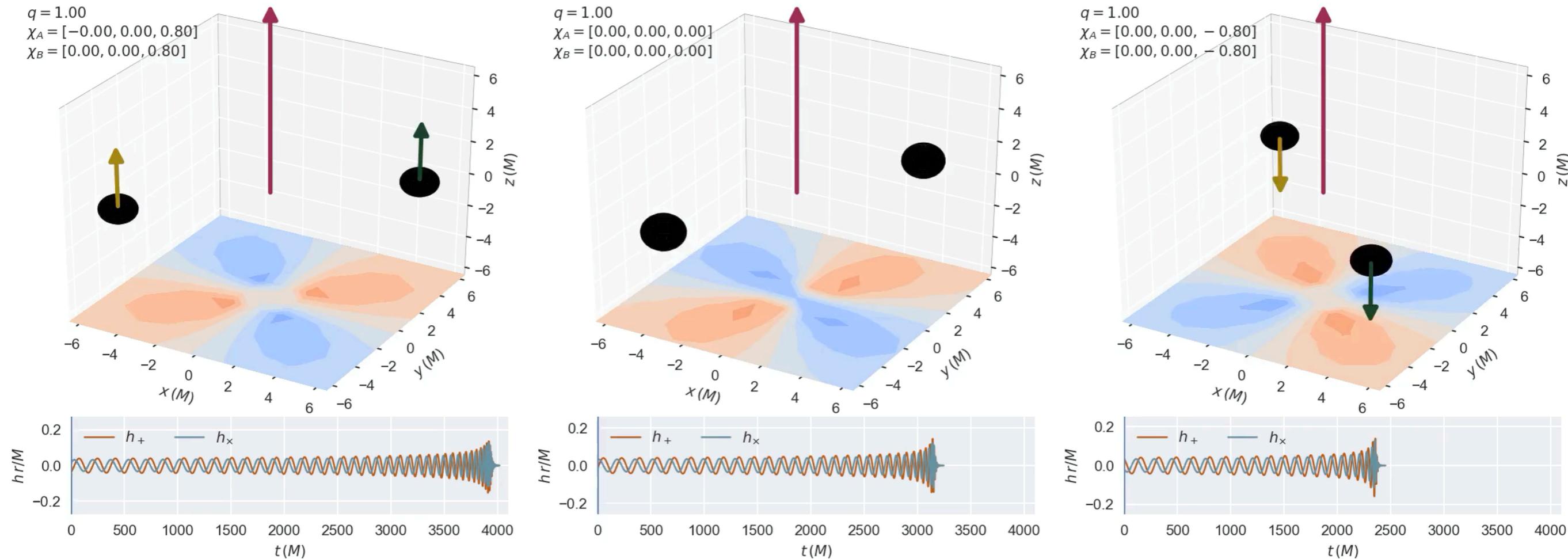


# Junk radiation



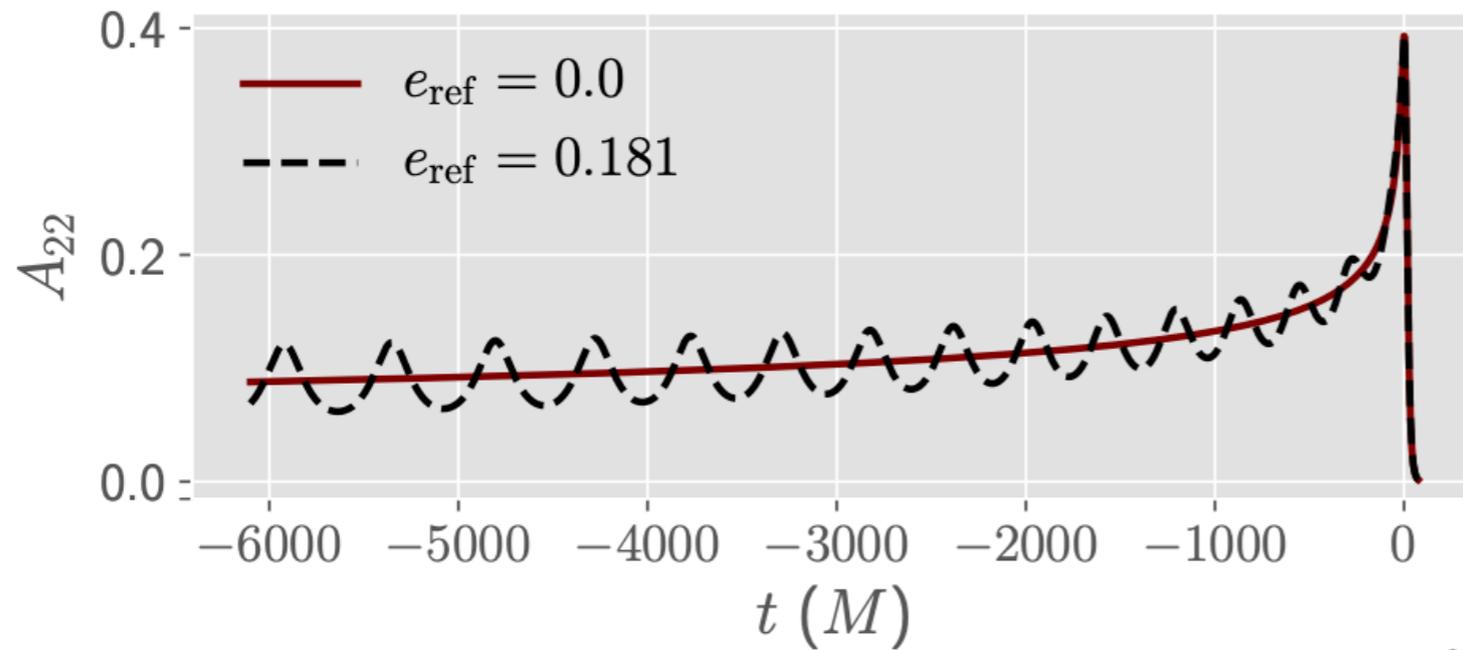


# Effective spin and orbital “hang-up” effect



- **Effective spin:**  $\chi_{\text{eff}} = (m_1 \chi_1 + m_2 \chi_2) / (m_1 + m_2)$ .
- **Binaries with positive  $\chi_{\text{eff}}$  merge at smaller separations, and at higher frequencies.**

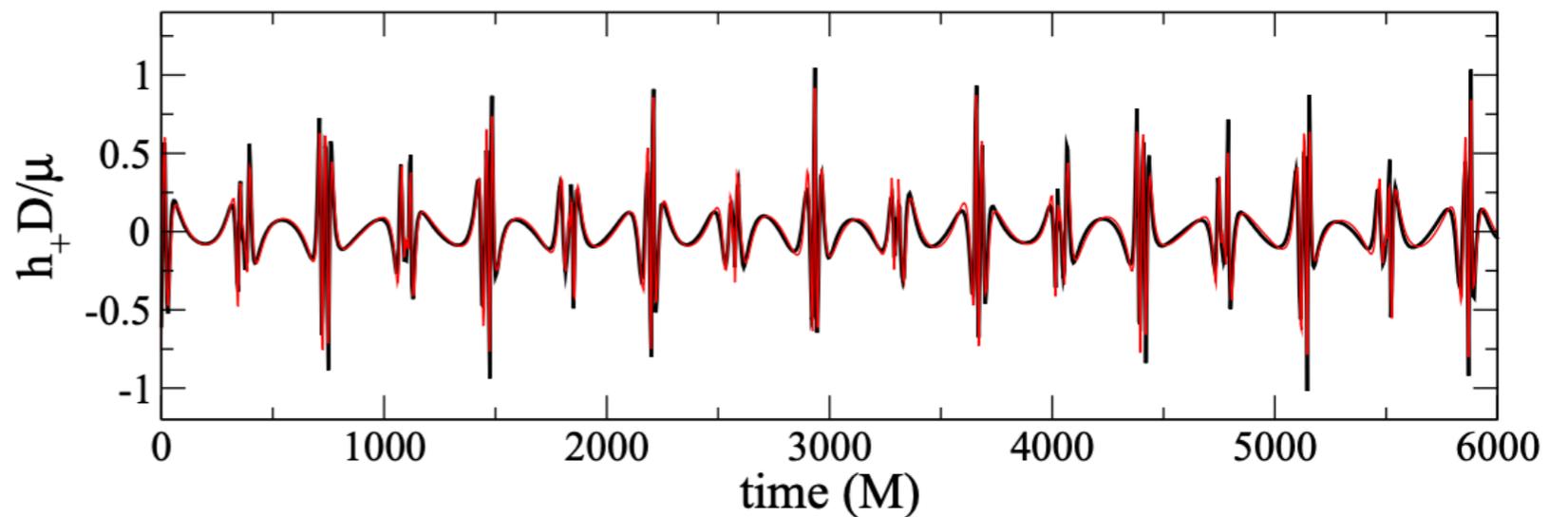
# Eccentricity



Islam+ (arxiv: 2101.11798)

$a = 0.9M, p = 6M, e = 0.7, \iota = 60(\text{deg}), \theta_d = 90(\text{deg})$

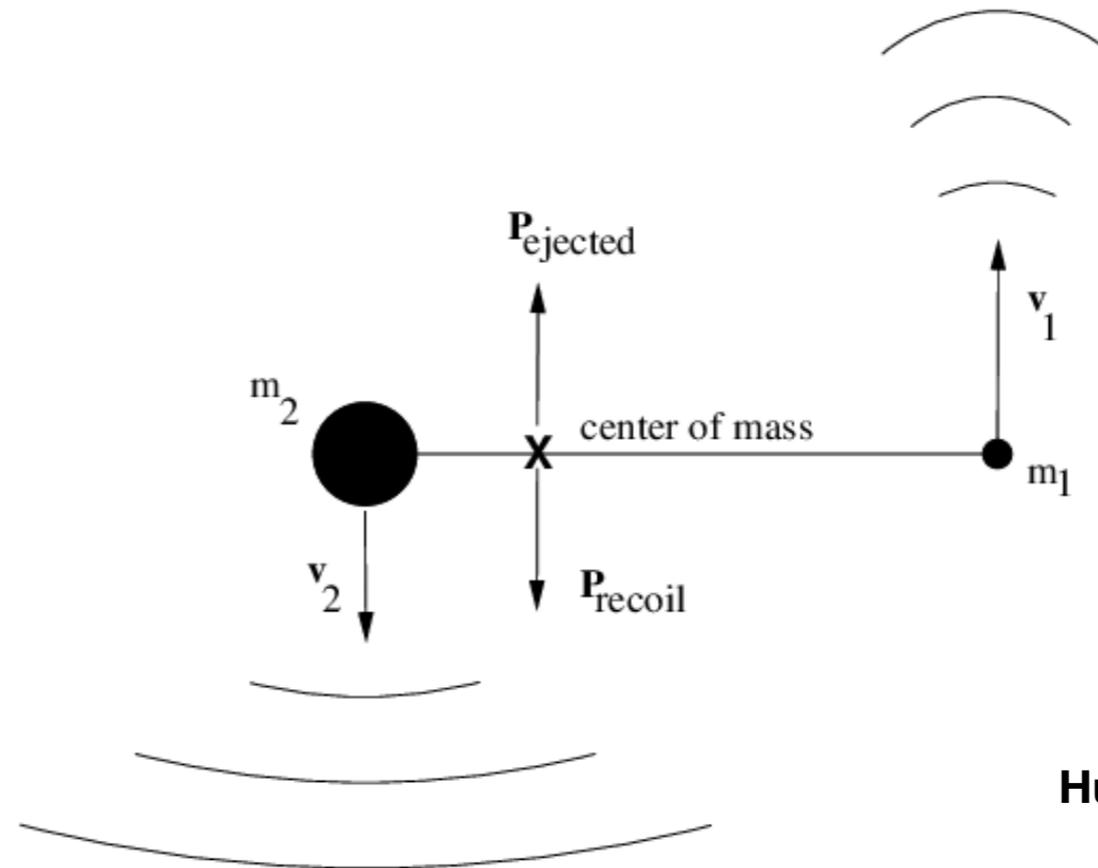
Babak+ (arxiv: 0607007)



- **Two additional parameters: eccentricity and mean anomaly.**
- **More important for LISA than LIGO.**

# Merger kicks

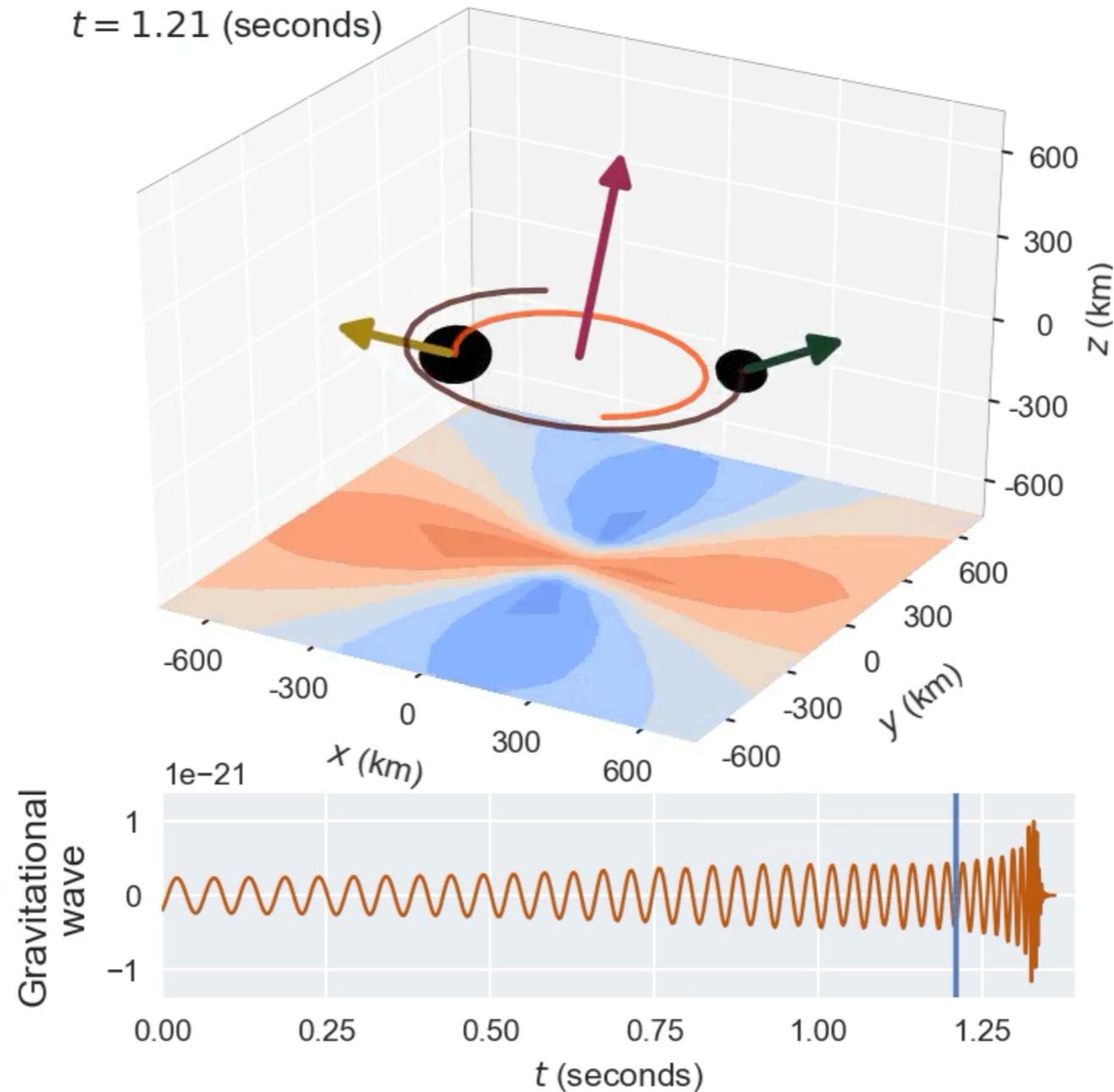
Asymmetry (in masses or spins)  
is necessary for kicks



Hughes+ (arxiv:0408492)

- **GWs can carry linear momentum away from BBHs**
- **Center-of-mass recoils to compensate**
- **Recoil or “kick” velocity imparted to final BH**
- **Kicks up to 5000 km/s possible for precessing BBHs**

# Superkicks



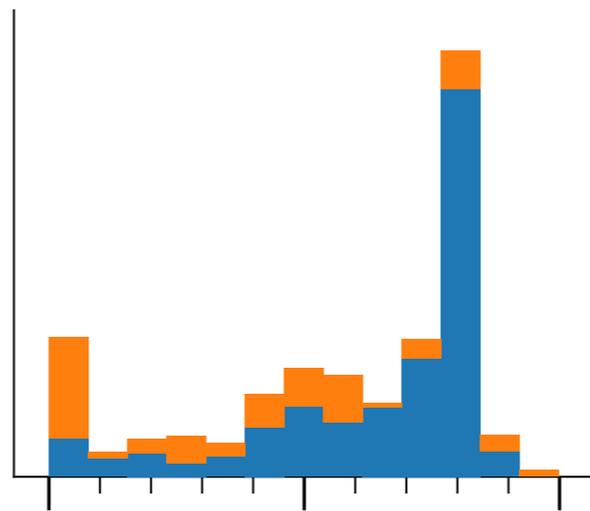
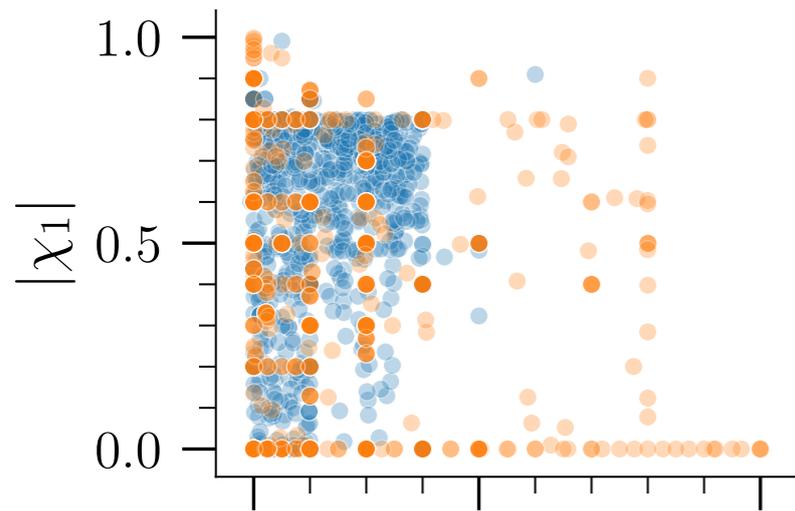
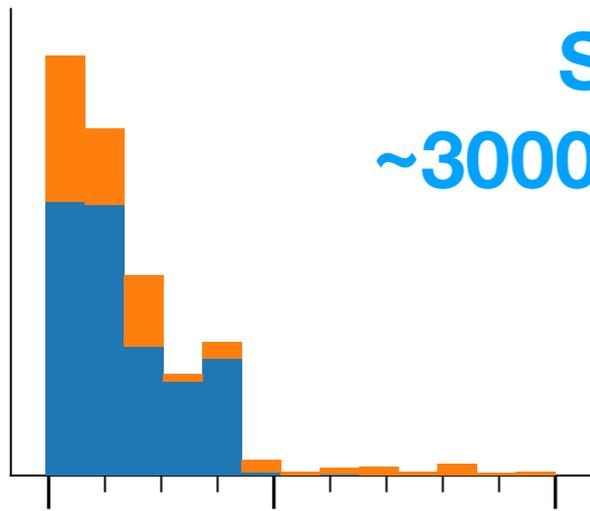
- **Spin asymmetry much more important than mass asymmetry.**

# Part III: Status of the field

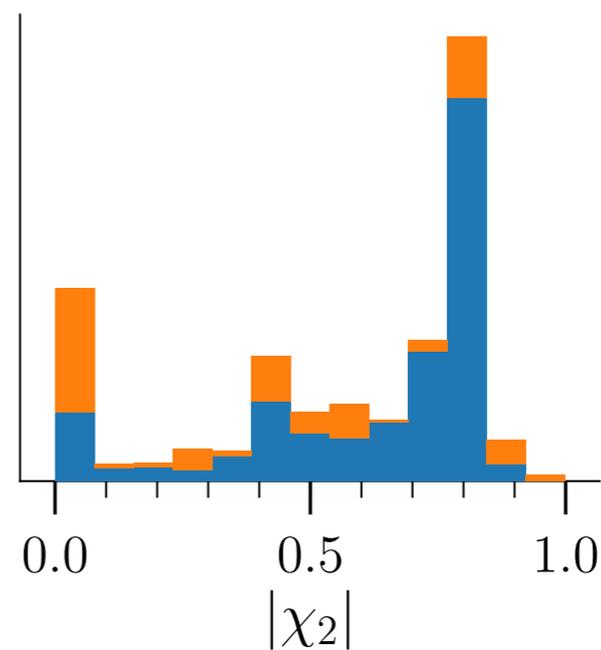
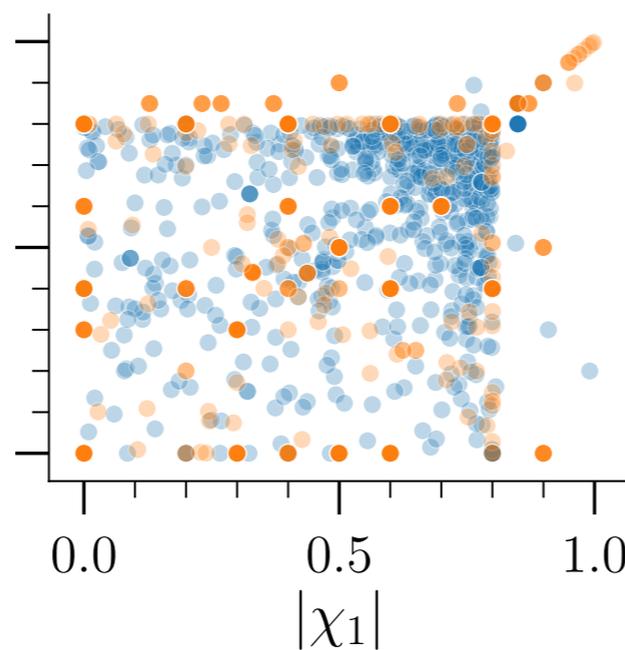
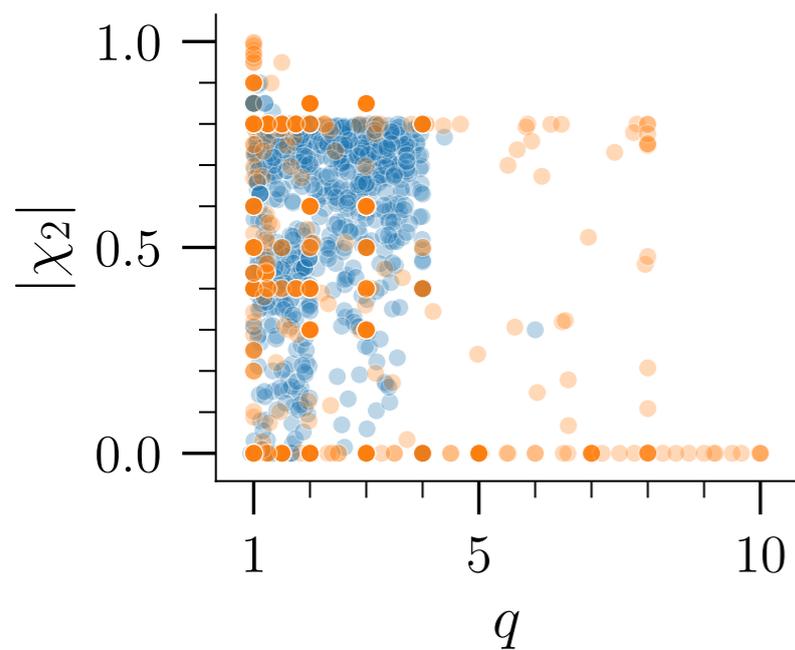
# NR simulations today

**Spectral Einstein Code (SpEC)**  
**~3000 simulations of binary black holes!**

- non-precessing  $\chi_{1x} = \chi_{1y} = \chi_{2x} = \chi_{2y} = 0$
- precessing

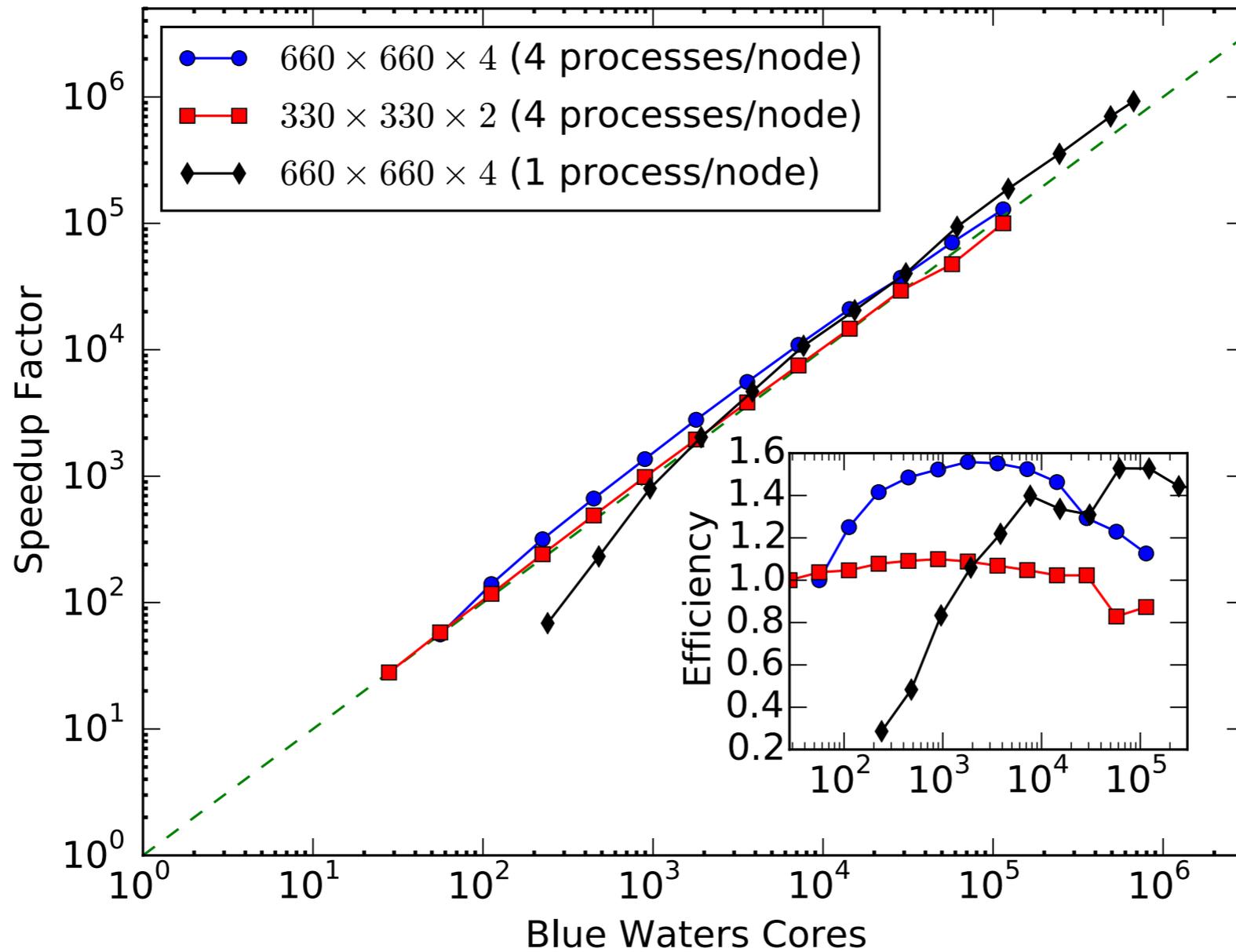


**SXS Collaboration, arXiv:1904.04831**



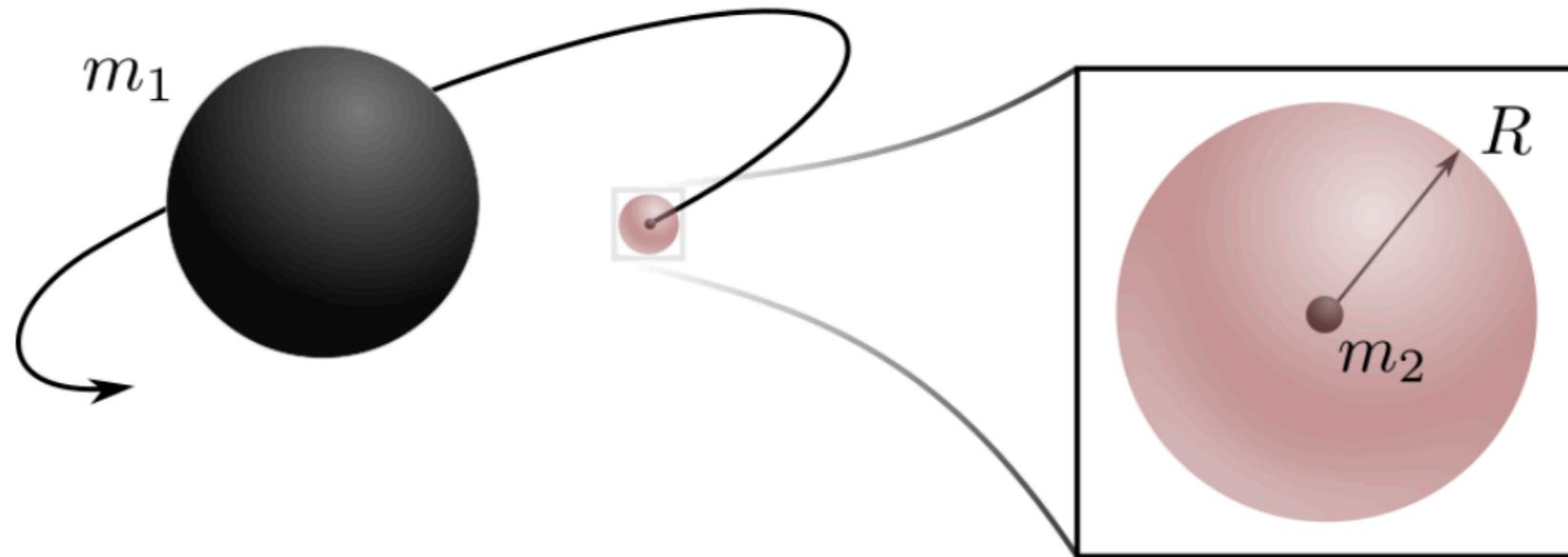
# Frontiers: Next generation NR codes

## SpECTRE



Kidder+ (1609.00098)

# Frontiers: Extreme mass ratios



Dhesi+ (2109.03531)

- Need to resolve the dynamics of the small BH.
- Maximum time steps  $\sim m_2$ , due to Courant condition.
- Talk to Niko.
- Near extremal spins: Scheel, Giesler+ (1412.1803)

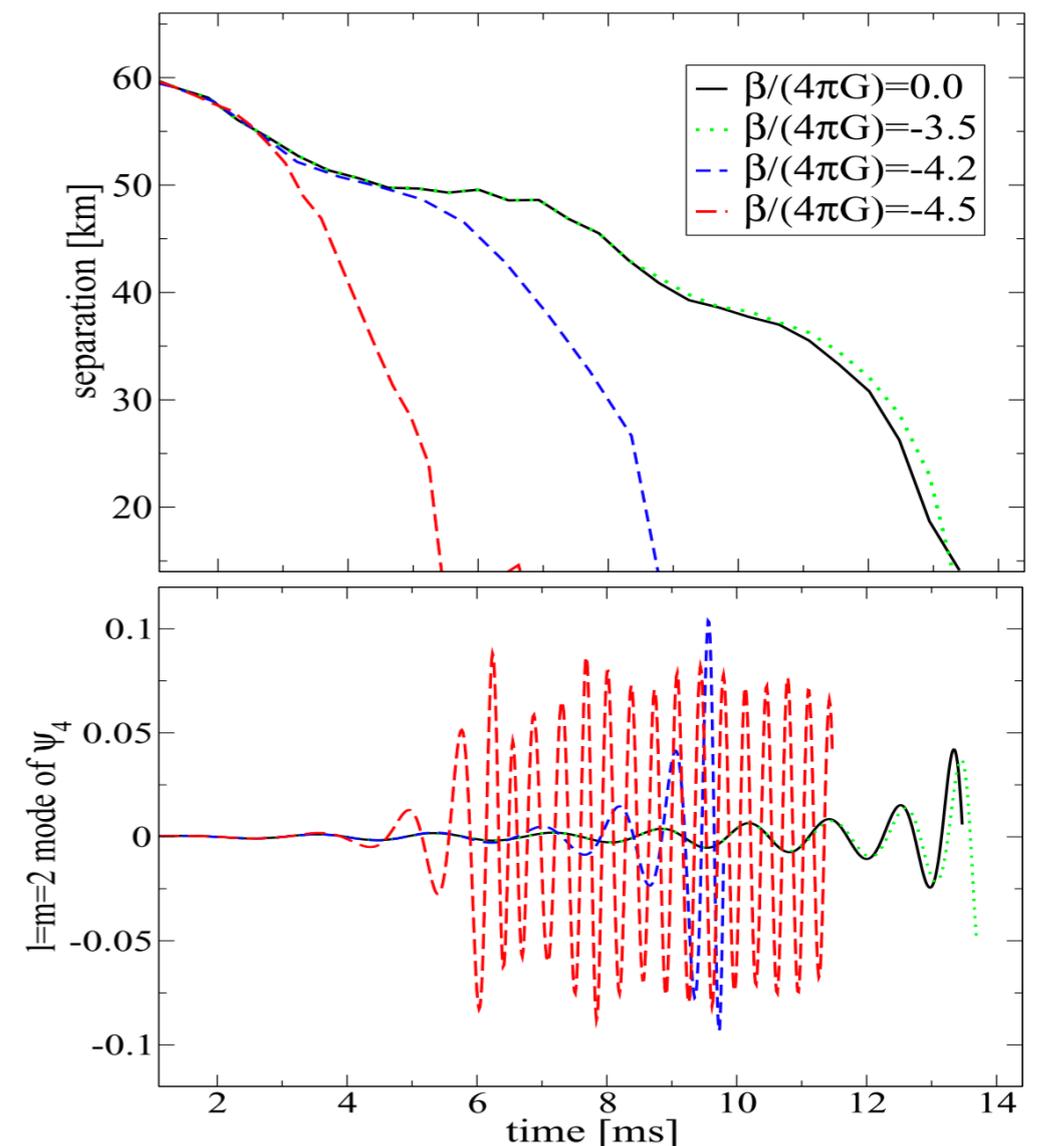
# Frontiers: Simulations of alternate theories

- BBHs in DCS, EdGB: Okounkova, Ripley, Witek, Silva, etc.
- BNS in scalar-tensor gravity: Barausse, Shibata, etc.
- BHNS in scalar-tensor gravity: Ma+Varma, etc.

## Challenges

- Numerical stability (well-posed?).
- Dealing with 3rd/4th derivatives of metric.
- Large parameter space, extra fields.
- Degeneracies with GR parameters.

## BNS merger in scalar-tensor



Barausse+ (1212.5053)