BH HORIZONS

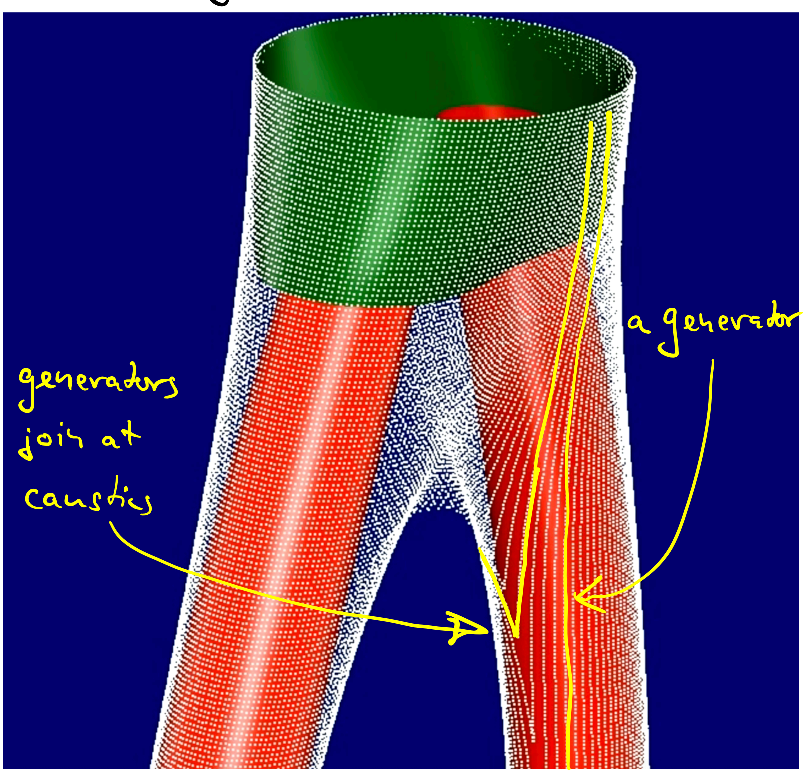
- Event horizons (global. Require 4-D data)
- Apparent horizons (local in time: Require 3-D data at t=const)

Event-Horizons

- foliated by null geodesics ("generators" of Horizon)
- which geodesics determined at late time
  - informally: those that don't fall into BH and neither escape
  - formally: boundary of past domain of dependence of $\mathcal{J}^+$
- new geodesics can enter horizon at caustics
- A_{EH} grows when geodesics diverge

Merger of two BHs "pair of pants"

space-time diagram



Key concept: <u>expansion</u> of null-geodesic congruence.

$$\Theta = \nabla_\mu k^\mu$$

Raychaudhuri's eqn for null congruences

$$\mathcal{L}_k \Theta = -\frac{1}{2}\Theta^2 - \sigma^{\mu\nu}\sigma_{\mu\nu} + \omega^{\mu\nu}\omega_{\mu\nu} - \underbrace{R_{\mu\nu}k^\mu k^\nu}_{\substack{=0 \text{ in vacuum} \\ \geq 0 \text{ for matter satisfying weak energy condition}}}$$

shear of congruence (= energy flux across)

twist of congruence $= 0$ b/c surface forming

$$\Rightarrow \frac{d\Theta}{d\lambda} \leq -\frac{1}{2}\Theta^2$$

affine param along geodesic

$\Theta < 0 \Rightarrow \Theta \to -\infty$ in finite affine parameter

$\Rightarrow$ Penrose singularity thm $\Rightarrow$ not on EH

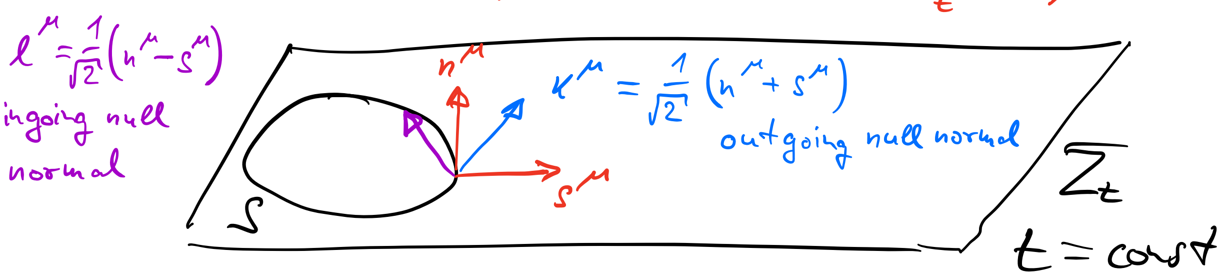
$\Rightarrow \Theta \geq 0$ on EH

$\Theta = 0$ on EH w/ constant area (non-growing)

Apparent Horizon

outermost marginally outer trapped surface MOTS

n^μ: unit normal $\perp$ to Σ_t $n^\mu n_\mu = -1$



s^μ: unit normal to S within Σ_t

$s^\mu s_\mu = +1$

null:

$$k_\mu k^\mu = \frac{1}{2}(n+s)(n+s) = \frac{1}{2}(\underbrace{n^2}_{-1} + \underbrace{2n\cdot s}_{0} + \underbrace{s^2}_{+1}) = 0$$

MOTS: $\Theta_{(k)} = \nabla_\mu k^\mu = 0$

In 3+1:

induced metric on Horizon:

$$m_{\mu\nu} = g_{\mu\nu} + k_\mu \ell_\nu + k_\nu \ell_\mu$$

$$\Theta_{(k)} = m^{\mu\nu} \nabla_\mu k_\nu$$

$$= \frac{1}{\sqrt{2}} m^{\mu\nu}(\nabla_\mu n_\nu + \nabla_\mu s_\nu)$$

$$= \frac{1}{\sqrt{2}} m^{\mu\nu}(-K_{\mu\nu} + \nabla_\mu s_\nu) \qquad (*)$$

all tensors in (*) are spatial, so can switch to spatial indices.

Moreover $m_{ij} = \gamma_{ij} - s_i s_j$

$$\Rightarrow \quad \Theta_{(k)} = \frac{1}{\sqrt{2}} \left(D_i s^i - K + K_{ij} s^i s^j \right) \qquad (**)$$

$\therefore$ to find AH (or more generally any MOTS), find topologically spherical surface S within Σ_t that satisfies $\Theta_{(k)} = 0$ everywhere on S.

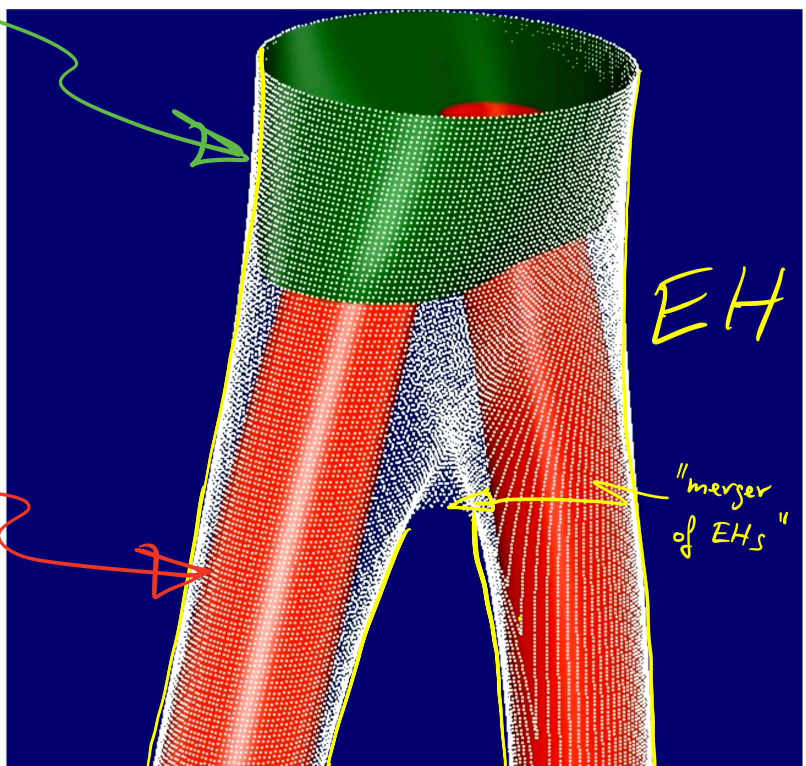
AH (MOTS) depend <u>only</u> on data on Σ_t (γ_{ij}, K_{ij}).

For stationary BHs (where EH has $\Theta = 0$), AH = EH

For growing BHs (where EH has $\Theta > 0$), $\underset{\Theta=0}{\text{AH}}$ inside $\underset{\Theta>0}{\text{EH}}$

Common AH

appears discontinuously

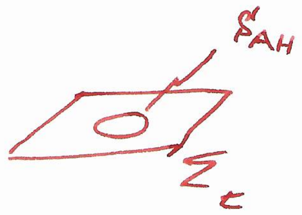


AHs of individual BHs

BH Mass + Spin

Horizon Area

$$A_{AH} = \int_{S_{AH}} dA$$



Angular Momentum (0805.4192 Appendix)

in axi-symmetry w/ rotational Killing-Vector ϕ^i: Komar integral

$$J = \frac{1}{8\pi} \int_{S'} (K_{ij} - g_{ij} K)\, \phi^i s^j \, dA \qquad (1)$$

- $S_\infty \Rightarrow J_{ADM}$
- $S'_{AH} \Rightarrow S_{BH}$

w/o axi-symmetry

- find approx rotational KV

$$\phi_i \text{ tangent to } S' \Rightarrow \phi^A = \epsilon^{AB} D_B z$$

$$\int_{S'} D_{(i}\phi_{j)} D^{(i}\phi^{j)} dA \to \min \Rightarrow \text{generalized Eigenvalue problem} \quad H z = \lambda D^2 z$$

- solve for ϕ_i^i then use in (1).

Spin Axis   Owen et al 1708.07325

simple: line connectin min $\to$ max of z

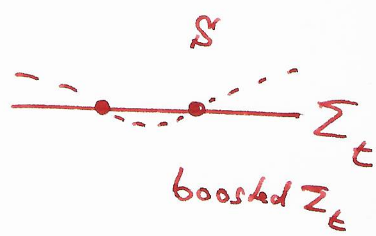
or

just use flat space rotation vectors in (1)

$$\phi_z = x\partial_y - y\partial_x \qquad \phi_x = y\partial_z - z\partial_y \qquad \phi_y = \dots$$

better: moments of z (Eq 27 of 1708.07325)

best: also make boost invariant (Eq 48)



BH Mass

Assume relation that holds for isolated Kerr

$$M^2 = M_{irr}^2 + \frac{S^2}{4 M_{irr}^2} \, , \quad M_{irr} = \sqrt{\frac{A}{16\pi}}$$

"Christodoulou formula"

BH Spin

$$\chi := \frac{S}{M^2}$$

note: by definition of M, must have $\underline{\chi \leq 1}$. (0805.4192, Eq 9)