Overview of BBH results in NR

Max Corman
Computational Physics IMPRS Course
June 27, 2025

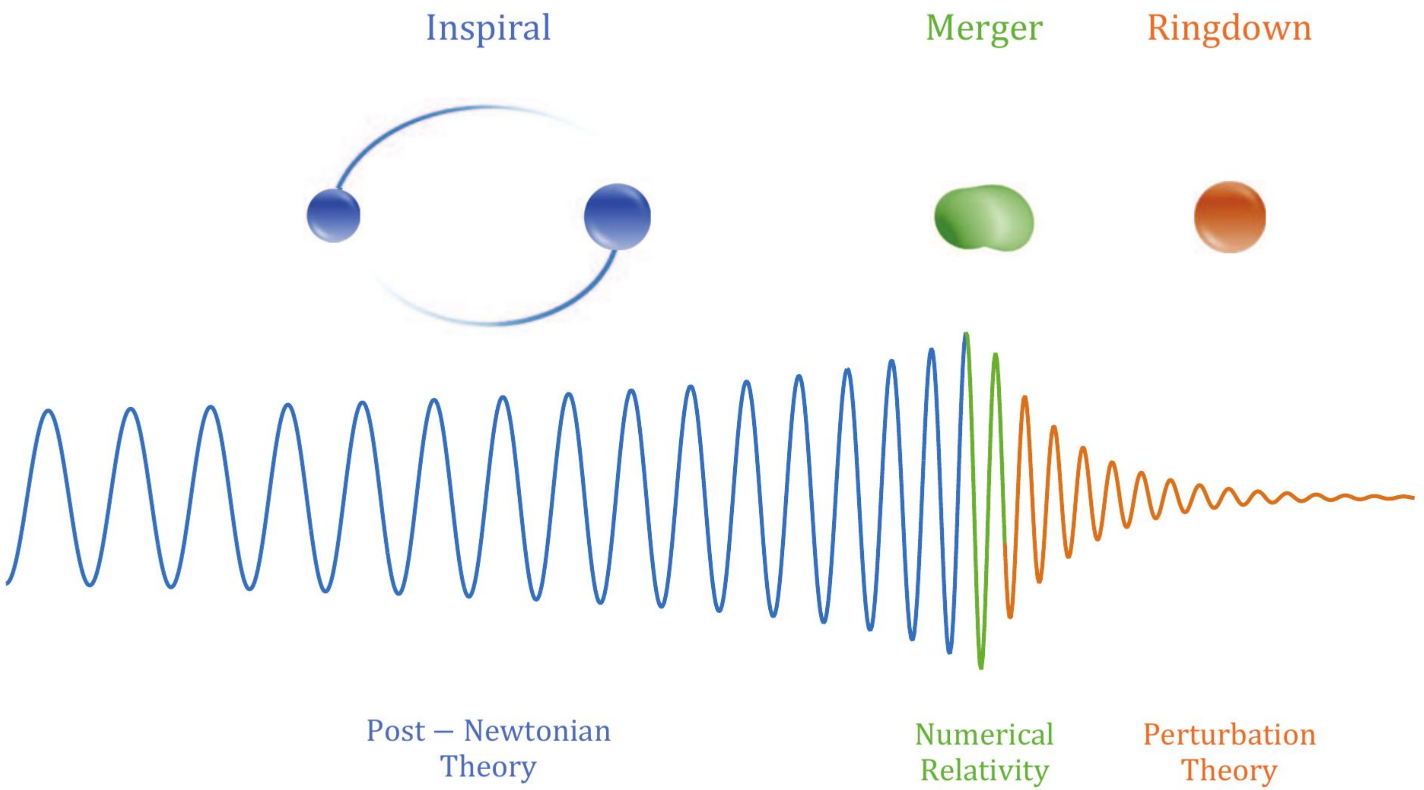
Credit: Nils Vu

Outline

- Brief history of NR
- Import BBH phenomena
  - ❏ Higher order modes
  - ❏ Junk radiation
  - ❏ Precession
  - ❏ “Hang up” effect
  - ❏ Eccentricity
  - ❏ Kicks
- Status of the field and frontiers

Part 1: Brief history of NR

Stages of a binary black hole inspiral



The very beginning

ANNALS OF PHYSICS: 29, 304–331 (1964)

The Two-Body Problem in Geometrodynamics

SUSAN G. HAHN

International Business Machines Corporation, New York, New York

AND

RICHARD W. LINDQUIST

The numerical calculations were carried out on an IBM 7090 electronic computer. The parameters a and μ_0 were both set equal to unity; the mesh lengths were assigned the values $h_1 = 0.02$, $h_2 = \pi/150 \approx 0.021$, yielding a 51 × 151 mesh. The calculations of all unknown functions, including a great number of input-output operations and some built-in checking procedures, took approximately four minutes per time step. Different check routines indicated that results close to the point $\mu = 0$, $\eta = 0$ lost accuracy fairly quickly. Since these would, in the long run, influence meshpoints further away, the computations were stopped after the 50th time step, when the total time elapsed was approximately 1.8. Some of the results are shown in Table I.

Evolution time 1.8 M !

40 years later...

VOLUME 92, NUMBER 21 PHYSICAL REVIEW LETTERS week ending 28 MAY 2004

Numerical Simulation of Orbiting Black Holes

Bernd Brügmann, Wolfgang Tichy, and Nina Jansen

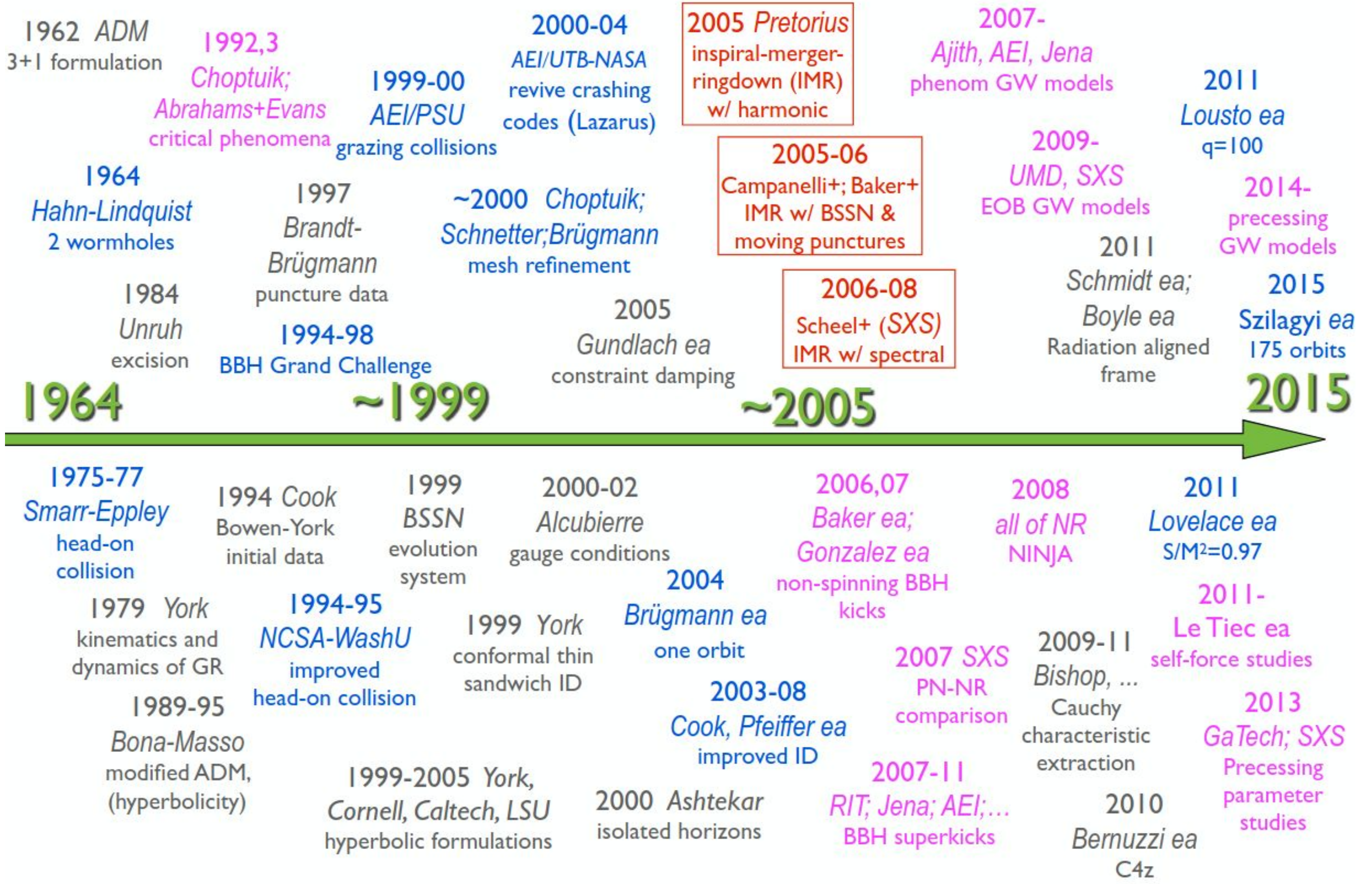
Center for Gravitational Physics and Geometry and Center for Gravitational Wave Physics, Penn State University, University Park, Pennsylvania 16802, USA

(Received 26 December 2003; published 24 May 2004)

We present numerical simulations of binary black hole systems which for the first time last for about one orbital period for close but still separate black holes as indicated by the absence of a common apparent horizon. An important part of the method is the construction of comoving coordinates, in which both the angular and the radial motion are minimized through a dynamically adjusted shift condition. We use fixed mesh refinement for computational efficiency.

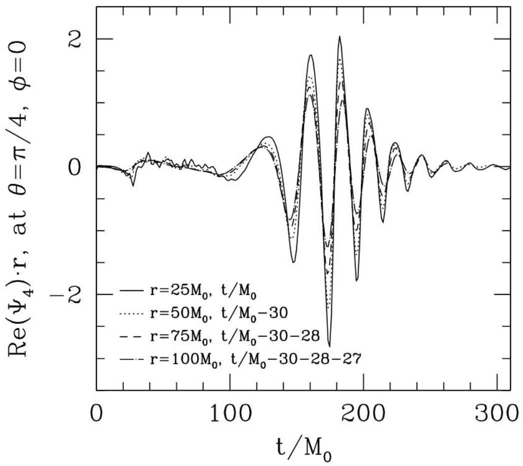
DOI: 10.1103/PhysRevLett.92.211101 PACS numbers: 04.25.Dm, 04.30.Db, 95.30.Sf

First 50 years of numerical relativity for BBH

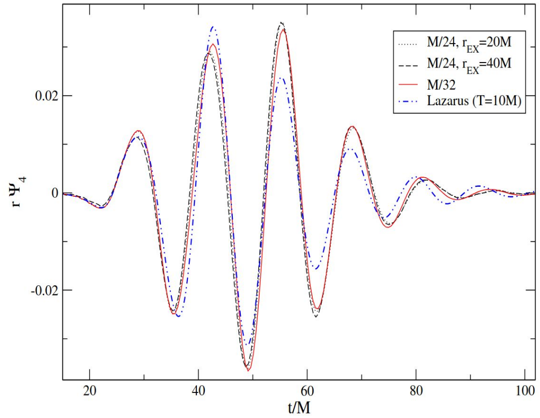


Credit: C. Lousto and H.Pfeiffer

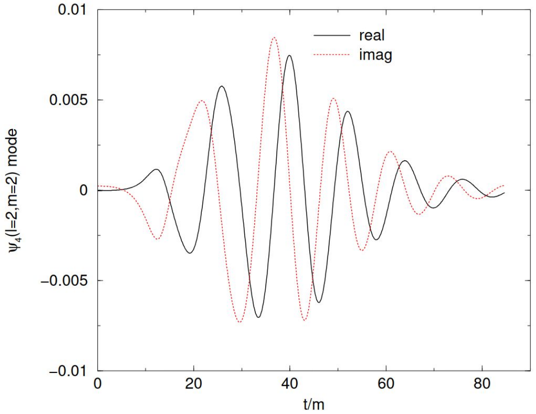
First BBH simulations



Pretorius 2005



Baker+ 2006



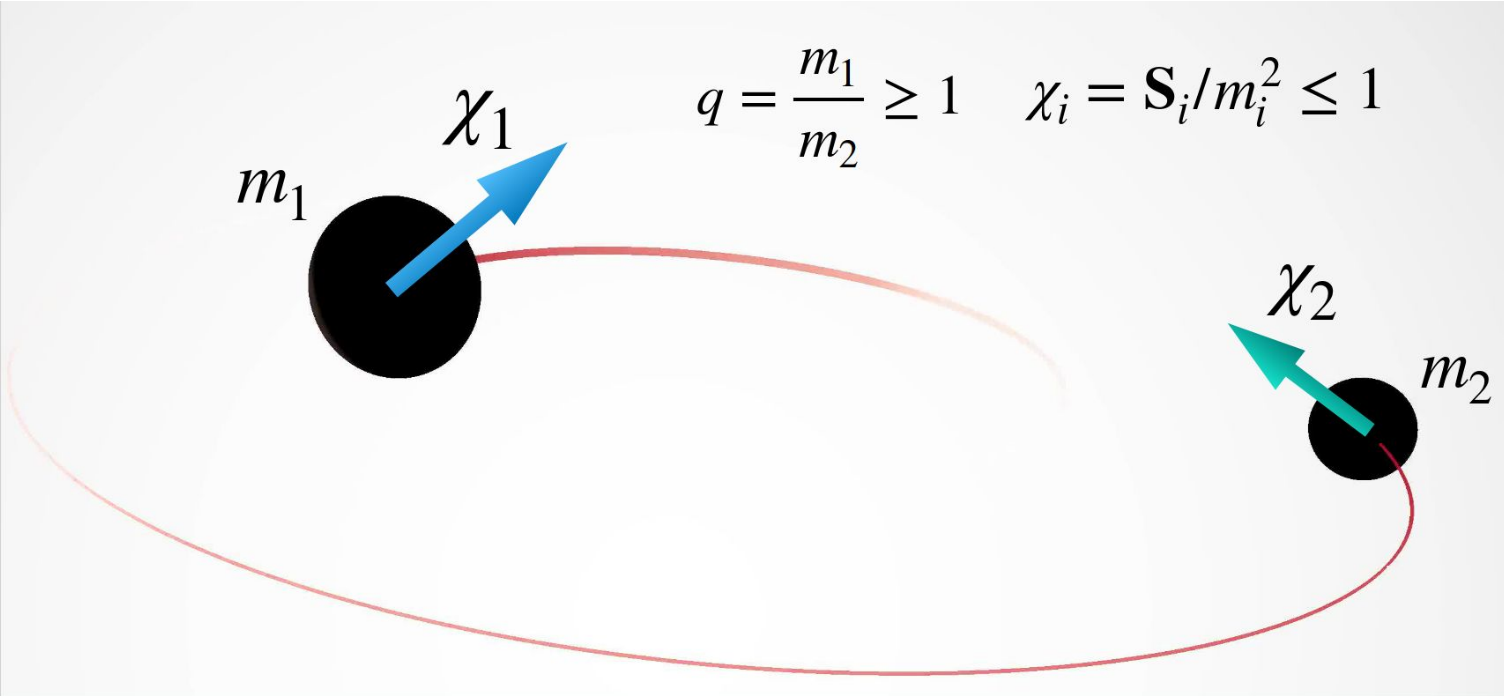
Campanelli+ 2006

Important early result:
Simplicity of merger

Continuous transition
inspiral to ringdown

Part 2: Important BBH phenomena

Binary black hole parameters



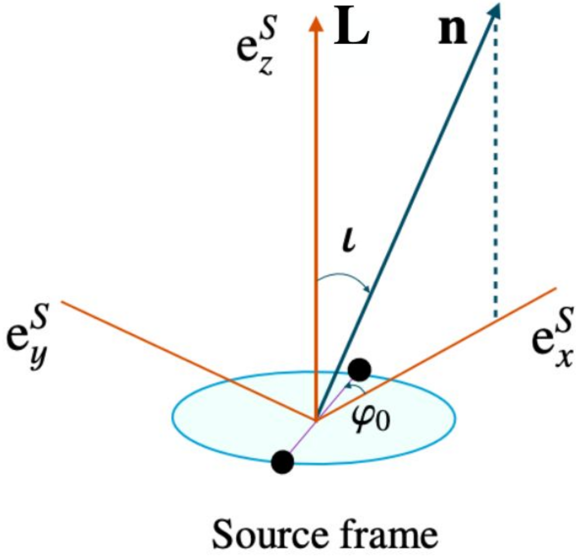
$$\Lambda = \{q, \chi_{1x}, \chi_{1y}, \chi_{1z}, \chi_{2x}, \chi_{2y}, \chi_{2z}\}$$

Higher order modes

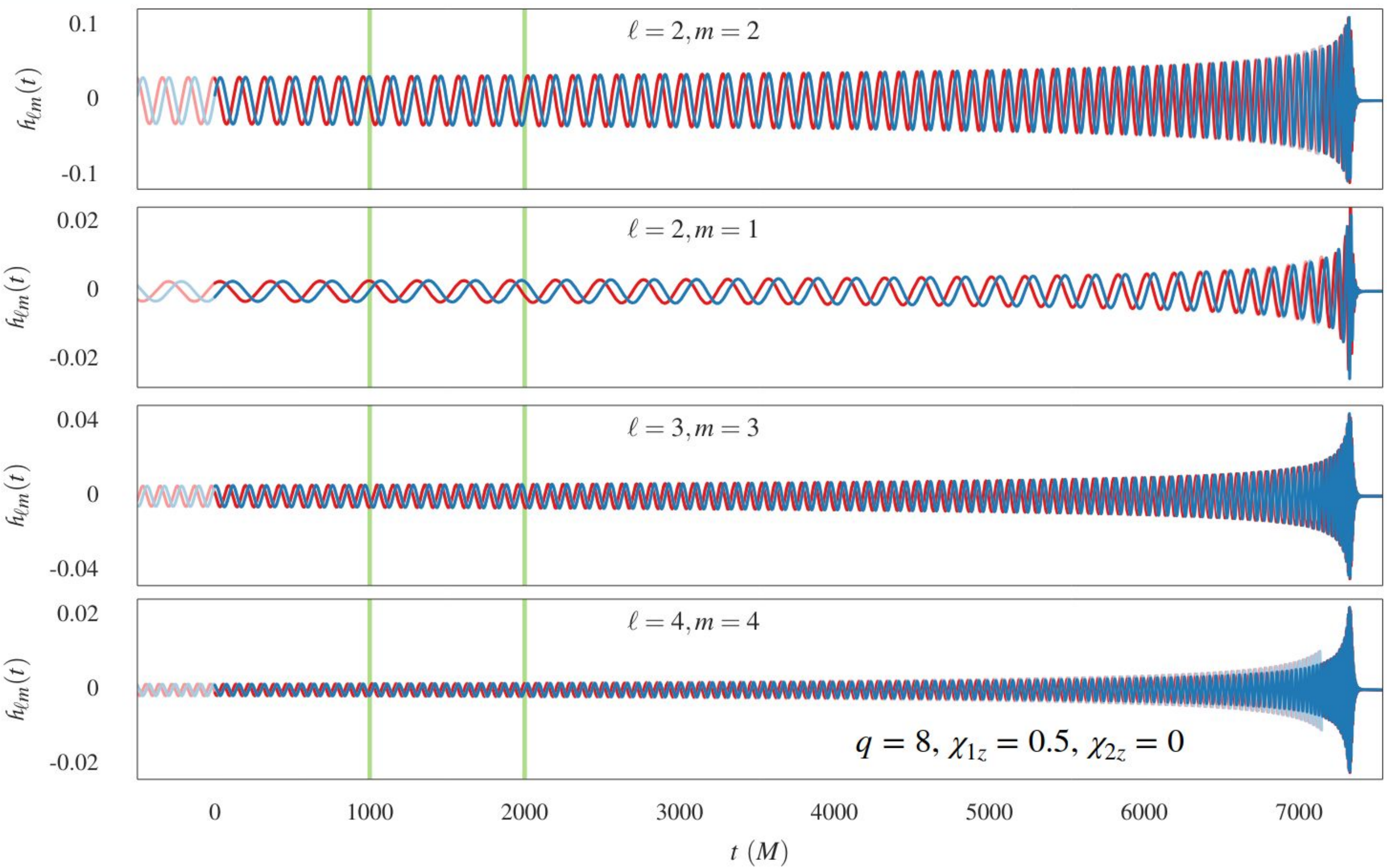
$$h = h_+ - i\, h_\times$$

$$h(\iota, \varphi_0, t) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} {}_{-2}Y_{\ell m}(\iota, \varphi_0)\; h_{\ell m}(t)$$

$$h_{\ell m} = A_{\ell m}\, e^{i\phi_{\ell m}}\;; \quad \omega_{lm} = \frac{d\phi_{\ell m}}{dt} \sim m\, \omega_{\rm orb}$$

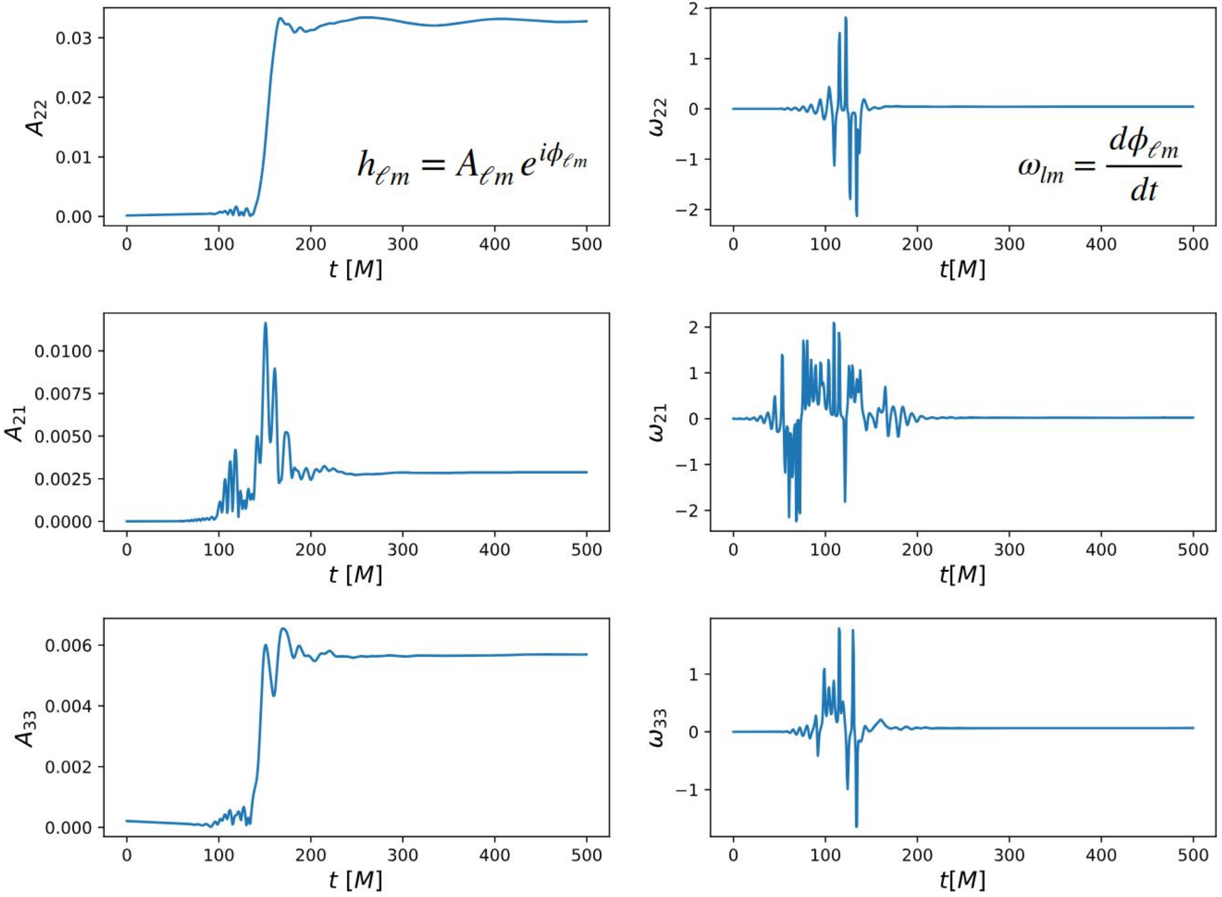


- Quadrupolar modes (2,+-2) are typical dominant
- Other modes are referred to as higher order modes, higher harmonics, nonquadrupolar modes or subdominant modes
- Higher order modes become important at large q or inclination angles



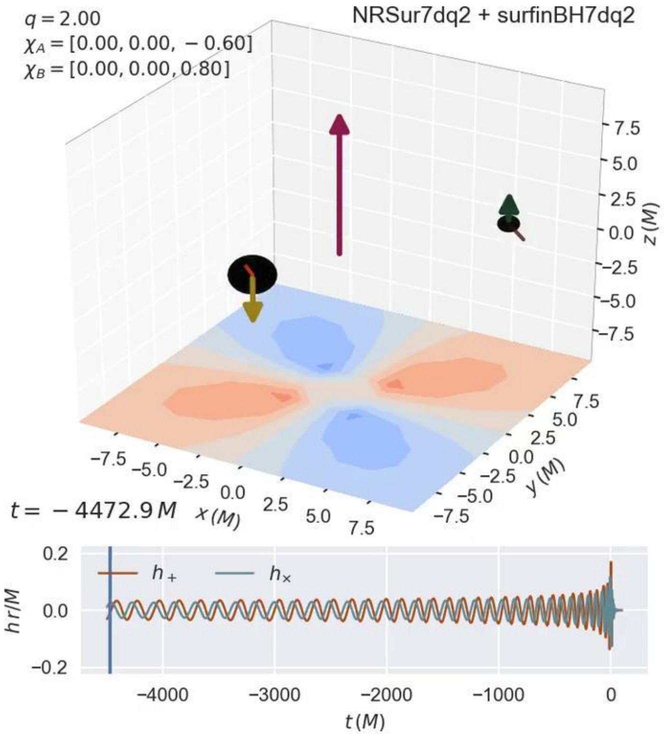
$\ell = 2, m = 2$
$\ell = 2, m = 1$
$\ell = 3, m = 3$
$\ell = 4, m = 4$
$q = 8,\ \chi_{1z} = 0.5,\ \chi_{2z} = 0$
$h_{\ell m}(t)$
0.1
0
-0.1
0.02
-0.02
0.04
-0.04
0
1000
2000
3000
4000
5000
6000
7000
$t\ (M)$

Junk radiation



Animations taken from:
https://vijayvarma392.github.io/binaryBHexp/#aligned_bbh

Precession



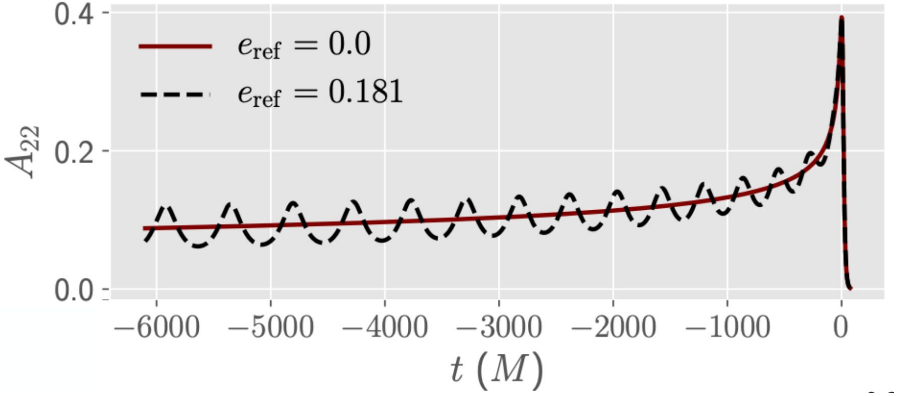
Aligned spin

Precessing

Effective spin and orbital hang up effect

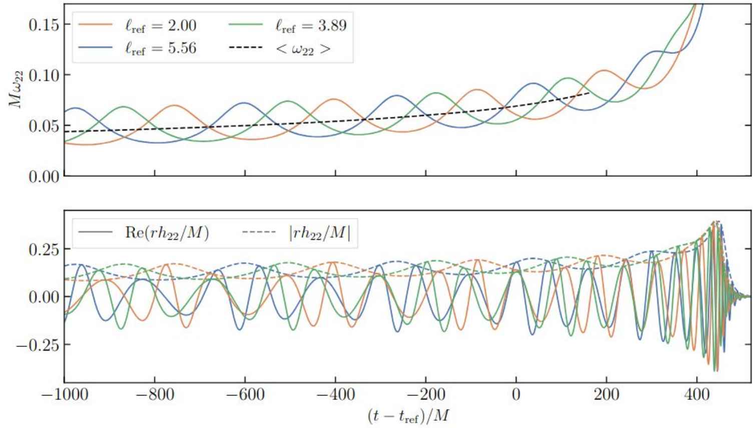
- Effective spin: $\chi_{\text{eff}} = (m_1 \chi_1 + m_2 \chi_2)/(m_1 + m_2)$
- Binaries with positive effective spin merge at smaller separation

Eccentricity

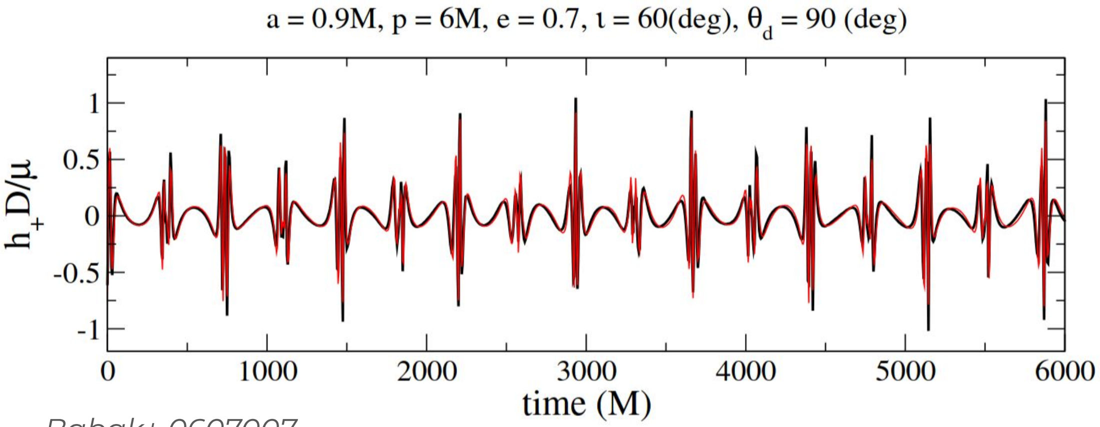


Islam+ 2101.11798

- Two additional parameters: eccentricity and mean anomaly
- More important for LISA than LIGO



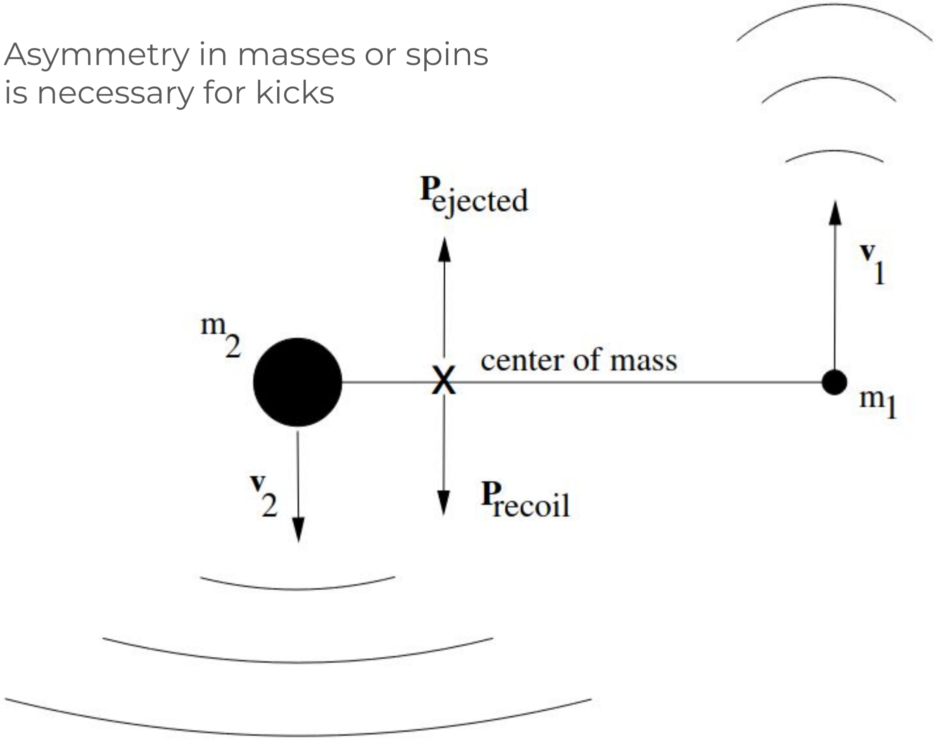
PJ Nee+ 2503.05422



Babak+ 0607007

Merger kicks

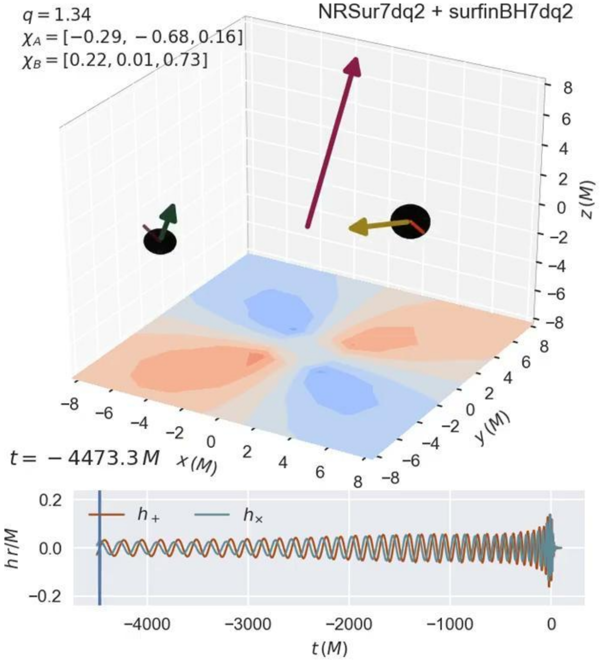
Asymmetry in masses or spins
is necessary for kicks



Hughes+ 0408492

- GWs can carry linear momentum away from BBHs
- Center-of-mass recoils to compensate
- Recoil or “kick” velocity imparted to final black hole
- Kicks up to 5000km/s possible for precessing BBHs

Super kicks



Spin asymmetry much more important than mass assymetry

Part 3: Status of the field and frontiers

NR codes today

Code	Open Source	Catalog	Formulation	Hydro	Beyond GR
AMSS-NCKU [43–46]	Yes	No	BSSN/Z4c	No	Yes
BAM [47–49]	No	[18]	BSSN/Z4c	Yes	No
BAMPS [50, 51]	No	No	GHG	Yes	No
COFFEE[52, 53]	Yes	No	GCFE	No	Yes
Dendro-GR [54–56]	Yes	No	BSSN/CCZ4	No	Yes
Einstein Toolkit [57, 58]	Yes	No	BSSN/Z4c	Yes	Yes
*Canuda [59–62]	Yes	No	BSSN	No	Yes
*IllinoisGRMHD [63]	Yes	No	BSSN	Yes	No
*LazEv [37, 64]	No	[65–68]	BSSN+CCZ4	No	No
*Lean [69, 70]	Partially	No	BSSN	No	Yes
*MAYA [71]	No	[71]	BSSN	No	Yes
*NRPy+ [72]	Yes	No	BSSN	Yes	No
*SphericalNR [73, 74]	No	No	spherical BSSN	Yes	No
*THC [75–77]	Yes	[18]	BSSN/Z4c	Yes	No
ExaHyPE [78]	Yes	No	CCZ4	Yes	No
FIL[79]	No	No	BSSN/Z4c/CCZ4	Yes	No
FUKA [80, 81]	Yes	No	XCTS	Yes	No
GR-Athena++ [82]	Yes	No	Z4c	Yes	No
GRChombo [83–85]	Yes	No	BSSN+CCZ4	No	Yes
HAD [86–88]	No	No	CCZ4	Yes	Yes
Illinois GRMHD [89, 90]	No	Yes	BSSN	Yes	No
MANGA/NRPy+ [91]	Partially	No	BSSN	Yes	No
MHDuet [92, 93]	No	No	CCZ4	Yes	Yes
SACRA-MPI [94]	No		BSSN+Z4c	Yes	No
SpEC [95, 96]	No	[96, 97]	GHG	Yes	Yes
SpECTRE [98, 99]	Yes	No	GHG	Yes	No
SPHINCS_BSSN [100]		No	BSSN	SPH	No

Foucart+. Snowmass2021 White paper, 2203.08193

NR simulations today

The SXS Collaboration's third catalog of binary black hole simulations

Mark A. Scheel[1], Michael Boyle[2], Keefe Mitman[2], Nils Deppe[2], Leo C. Stein[3], Cristóbal Armaza[2], Marceline S. Bonilla[4], Luisa T. Buchman[5], Andrea Ceja[4], Himanshu Chaudhary[1], Yitian Chen[2], Maxence Corman[6], Károly Zoltán Csukás[7], C. Melize Ferrus[8], Scott E. Field[9], Matthew Giesler[2], Sarah Habib[1], François Hébert[1], Daniel A. Hemberger[1], Dante A. B. Iozzo[2], Tousif Islam[10,9], Ken Z. Jones[4], Aniket Khairnar[3], Lawrence E. Kidder[2], Taylor Knapp[1], Prayush Kumar[11], Guillermo Lara[6], Oliver Long[6], Geoffrey Lovelace[4,1], Sizheng Ma[1], Denyz Melchor[4], Marlo Morales[4], Jordan Moxon[1], Peter James Nee[6], Kyle C. Nelli[1], Eamonn O'Shea[2], Serguei Ossokine[6], Robert Owen[12], Harald P. Pfeiffer[6], Isabella G. Pretto[1], Teresita Ramirez-Aguilar[4], Antoni Ramos-Buades[13], Adhrit Ravichandran[9], Abhishek Ravishankar[9], Samuel Rodriguez[4], Hannes R. Rüter[14], Jennifer Sanchez[4], Md Arif Shaikh[15], Dongze Sun[1], Béla Szilágyi[1], Daniel Tellez[4], Saul A. Teukolsky[2,1], Sierra Thomas[4], William Throwe[2], Vijay Varma[9], Nils L. Vu[1], Marissa Walker[4,16], Nikolas A. Wittek[6] and Jooheon Yoo[2]

(Affiliation list at end.)

2505.13378, **3756** SpEC simulations

The Fourth RIT binary black hole simulations catalog: Extension to Eccentric Orbits

James Healy and Carlos O. Lousto

Center for Computational Relativity and Gravitation,
School of Mathematical Sciences, Rochester Institute of Technology,
85 Lomb Memorial Drive, Rochester, New York 14623
(Dated: February 2, 2022)

This fourth release of the RIT public catalog of numerical relativity black-hole-binary waveforms http://ccrg.rit.edu/~RITCatalog consists of 1881 accurate simulations that include 446 precessing and 611 nonprecessing quasicircular/inspiraling binary systems with mass ratios $q = m_1/m_2$ in the range $1/128 \le q \le 1$ and individual spins up to $s/m^2 = 0.95$; and 824 in eccentric orbits in the range $0 < e \le 1$. The catalog also provides initial parameters of the binary, trajectory information, peak radiation, and final remnant black hole properties. The waveforms are corrected for the center of mass drifting and are extrapolated to future null infinity. As an application of this waveform catalog we reanalyze all of the peak radiation and remnant properties to find new, simple, correlations among them, valid in the presence of eccentricity, for practical astrophysical usage.

2202.00018, **1881** RIT simulations

Georgia Tech Catalog of Gravitational Waveforms

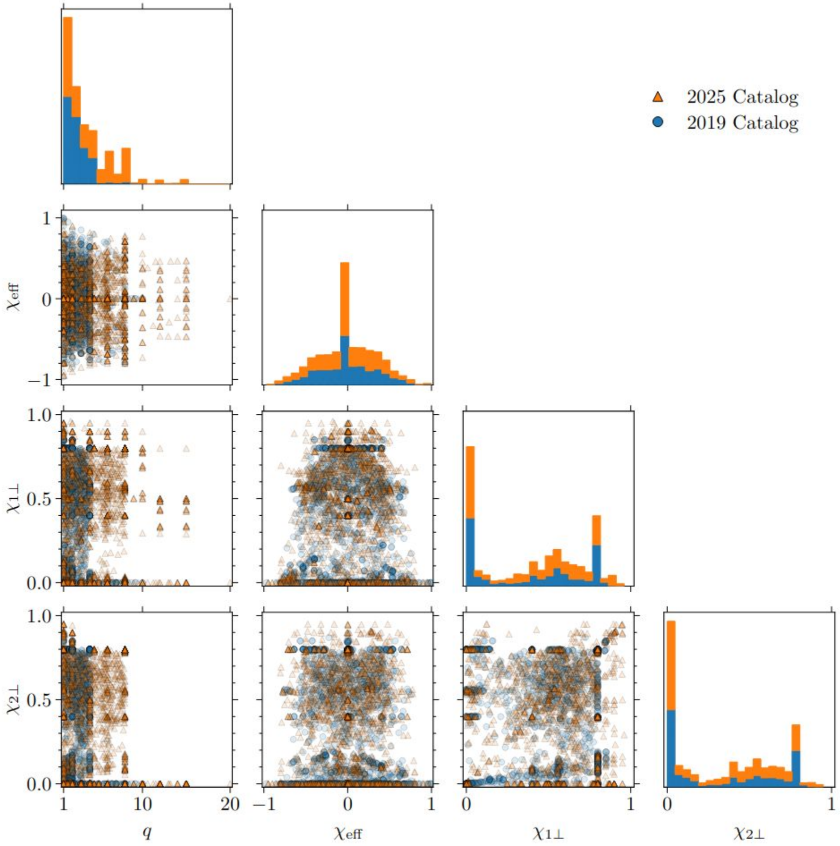
Karan Jani,[1] James Healy,[2,1] James A. Clark,[1] Lionel London,[3,1] Pablo Laguna,[1] and Deirdre Shoemaker[1]

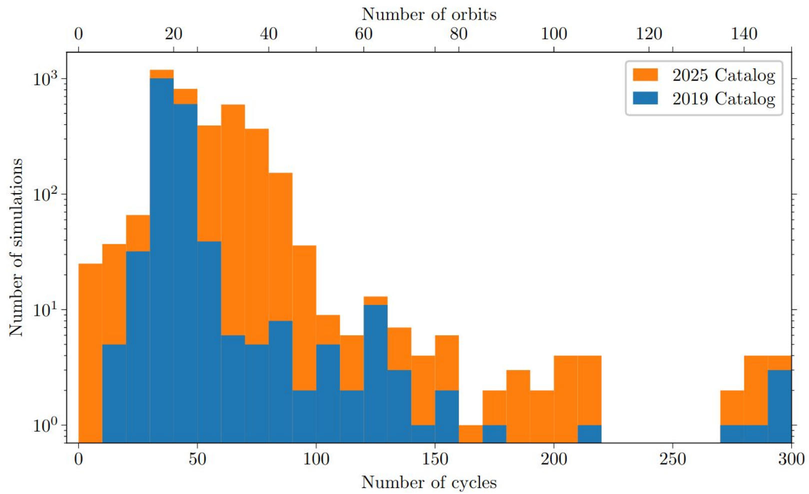
[1] *Center for Relativistic Astrophysics and School of Physics Georgia Institute of Technology, Atlanta, GA 30332*
[2] *Center for Computational Relativity and Gravitation Rochester Institute of Technology, Rochester, NY 14623*
[3] *School of Physics and Astronomy, Cardiff University Queens Building, CF24 3AA, Cardiff, United Kingdom*

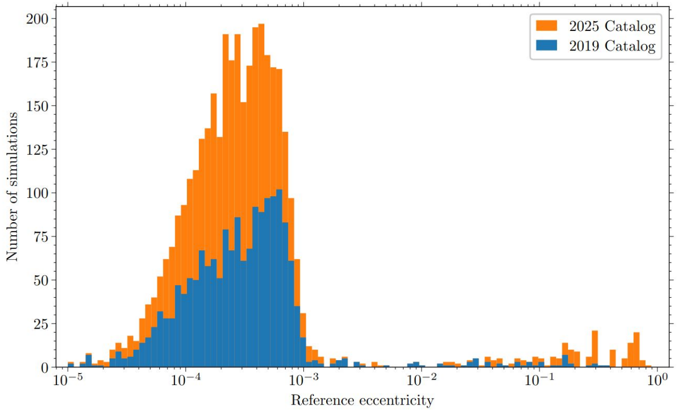
This paper introduces a catalog of gravitational waveforms from the bank of simulations by the numerical relativity effort at Georgia Tech. Currently, the catalog consists of 452 distinct waveforms from more than 600 binary black hole simulations: 128 of the waveforms are from binaries with black hole spins aligned with the orbital angular momentum, and 324 are from precessing binary black hole systems. The waveforms from binaries with non-spinning black holes have mass-ratios $q = m_1/m_2 \le 15$, and those with precessing, spinning black holes have $q \le 8$. The waveforms expand a moderate number of orbits in the late inspiral, the burst during coalescence, and the ring-down of the final black hole. Examples of waveforms in the catalog matched against the widely used approximate models are presented. In addition, predictions of the mass and spin of the final black hole by phenomenological fits are tested against the results from the simulation bank. The role of the catalog in interpreting the GW150914 event and future massive binary black-hole search in LIGO is discussed. The Georgia Tech catalog is publicly available at einstein.gatech.edu/catalog.

1605.03204, **452** Georgia tech simulations

SXS catalogue





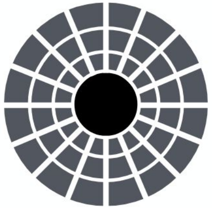


Frontiers: Next generation NR codes

Simulating eXtreme Spacetimes collaboration

s

Discontinuous Galerkin



Spectral

$$f(x) = \sum_{n=0}^{N} a_n \phi(x)$$

https://spectre-code.org

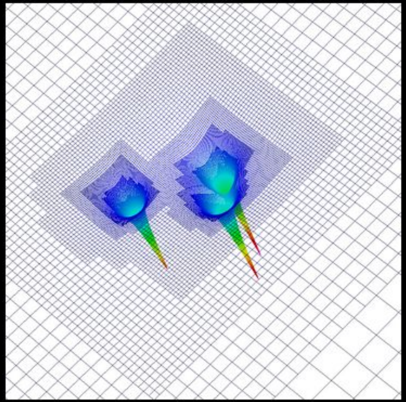
Performance-Portable Numerical Relativity with AthenaK

Hengrui Zhu (朱恒锐),[1,2] Jacob Fields,[3,4] Francesco Zappa,[5] David Radice,[3,6,4,*] James M. Stone,[7] Alireza Rashti,[3,4] William Cook,[5] Sebastiano Bernuzzi,[5] Boris Daszuta,[5]

[1] *Department of Physics, Princeton University, Jadwin Hall, Washington Road, New Jersey, 08544, USA*
[2] *Princeton Gravity Initiative, Princeton University, Princeton, New Jersey, 08544, USA*
[3] *Department of Physics, The Pennsylvania State University, University Park, PA 16802*
[4] *Institute for Gravitation & the Cosmos, The Pennsylvania State University, University Park, PA 16802*
[5] *Theoretisch-Physikalisches Institut, Friedrich-Schiller-Universität Jena, 07743, Jena, Germany*
[6] *Department of Astronomy & Astrophysics, The Pennsylvania State University, University Park, PA 16802*
[7] *School of Natural Sciences, Institute for Advanced Study, 1 Einstein Drive, Princeton, NJ 08540, USA*

ABSTRACT

We present the numerical relativity module within `AthenaK`, an open source performance-portable astrophysics code designed for exascale computing applications. This module employs the Z4c formulation to solve the Einstein equations. We demonstrate its accuracy through a series of standard numerical relativity tests, including convergence of the gravitational waveform from binary black hole coalescence. Furthermore, we conduct scaling tests on OLCF Frontier and NERSC Perlmutter, where `AthenaK` exhibits excellent weak scaling efficiency of 80% on up to 65,536 `AMD` MI250X GPUs on Frontier (relative to 4 GPUs) and strong scaling efficiencies of 84% and 77% on `AMD` MI250X and `NVIDIA` A100 GPUs on Frontier and Perlmutter respectively. Additionally, we observe a significant performance boost, with two orders of magnitude speedup ($\gtrsim 200\times$) on a GPU compared to a single CPU core, affirming that `AthenaK` is well-suited for exascale computing, thereby expanding the potential for breakthroughs in numerical relativity research.

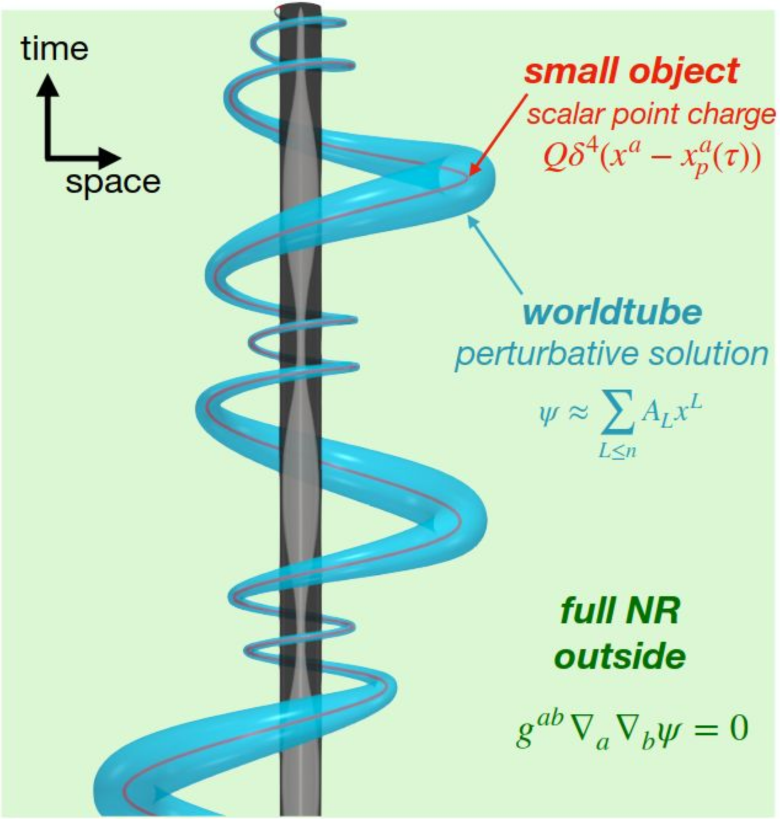


GRTeclyn

Coming soon! Our new numerical relativity code built on the AMReX framework, with GPU support

Key contacts:
Juliana Kwan, Miren Radia and Katy Clough

Frontiers:High mass ratio



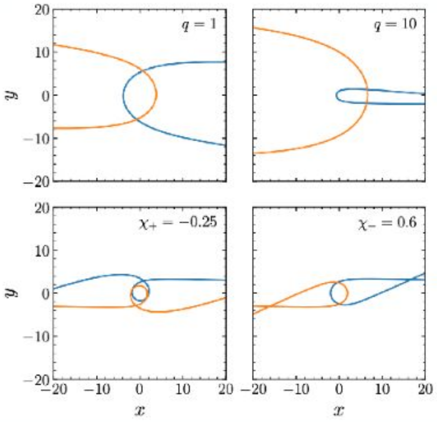
Credit: H.Peiffer

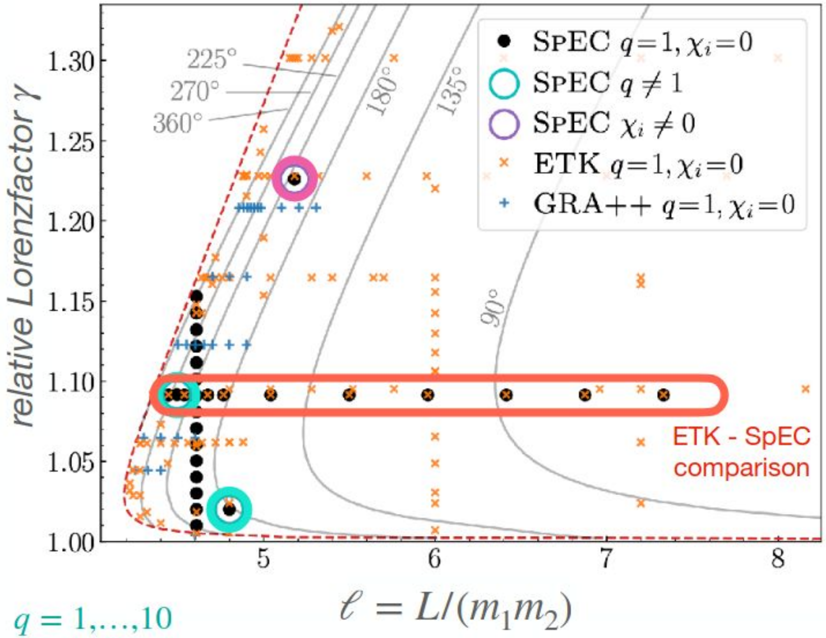
spectre

- Alleviates **Courant limit on time-step Δt**
  - $\Delta t \propto R_{\text{worldtube}}$
- So far test-problem: scalar point-charge around BH

Dhesi, Rüter+ PRD (2021)
Wittek, Dhesi..HP+ PRD (2023)
Wittek, Pound, Barack, HP PRD (2024)
Wittek, Barack..HP+ PRL (2025)

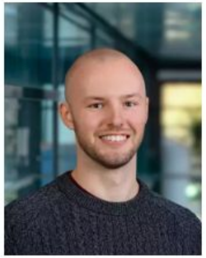
Frontiers:BH scattering





$q = 1,\ldots,10$

$\chi_{1,2} = -0.5\ldots0.5$



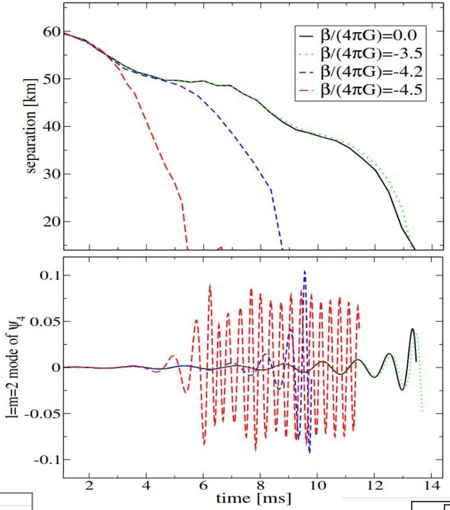
Long, *HP, Buonanno, Jacobson, Mogul+ in prep*

ETK
Damour, Guercilena, Hinder+ PRD (2014)
Hopper, Nagar, Rettegno PRD (2023) 2204.10299
Rettegno, Pratten, Thomas+ PRD (2023) 2307.06999
Swain,Pratten,Schmidt 2411.09652
GR-Athena++
Albanesi, Rashti, Zappa+ PRD (2025) 2405.20398

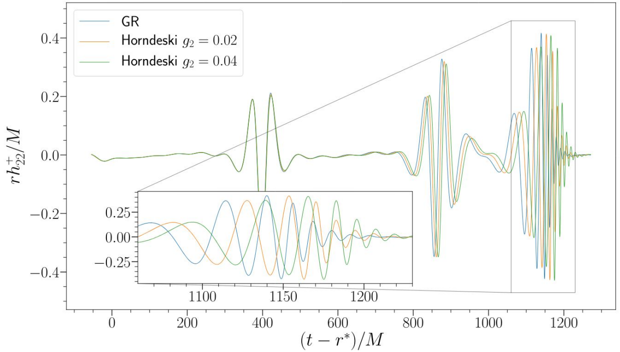
Frontiers: Simulations of alternative theories of gravity

Challenges:

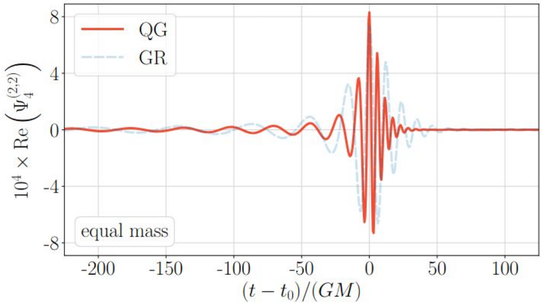
- Initial data **(PJ)**
- Well-posedness **(Max and Memo)**
- Large class of modified theories
- Degeneracies with GR



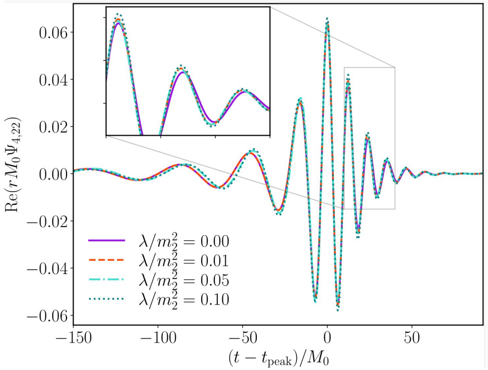
Scalar-tensor theories: Barausse+,Shibata+



Cubic Horndeski: Figueras+



Quadratic gravity: Held+,East+



Einstein scalar Gauss Bonnet: Corman+,East+,Salo+