Astrophysics and detection of gravitational wave sources

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OUTLINE

LECTURE 6/7 (Tuesday afternoon): space based/PTA -Massive black hole binaries (MBHBs): formation and dynamics -LISA science with MBHBs -Pulsar Timing Arrays (PTA): principles

LECTURE 8 (Wednesday morning): PTA -MBHB detection PTAs: status and prospects

*Physics of compact objects in GR and beyond (Prof. Gualtieri)

*Data analysis and GR tests (Prof. Del Pozzo)

*Multimessenger astronomy with GW and EM signals (Prof. Branchesi)

Massive black hole binaries

Cosmology in two slides

According to our best cosmological models, we live in a <u>ACDM Universe</u>. The energy content of the Universe is <u>27%</u> in the form of <u>ordinary matter</u> (~3% baryons, ~24% dark matter) and <u>73%</u> in the form of a <u>cosmological constant</u> (or Dark energy, or whatever), which would be responsible of the accelerated expansion.



The age of the Universe is ~14Gyr, during this time its size has expanded from a singularity to ~10²⁸cm.

Usually cosmologists describe the epochs of the Universe in terms of *redshift*:

$$z \equiv \frac{\nu_{\rm e}}{\nu_{\rm o}} - 1 = \frac{\lambda_{\rm o}}{\lambda_{\rm e}} - 1$$

which describe how much the photons emitted at a given time are redshifted, because of the expansion, when they arrive on the Earth.

The redshift of a photon is related to the size of the Universe at the moment of its emission through:

$$1 + z = \frac{a(t_{\rm o})}{a(t_{\rm e})}$$

A given redshift correspond to a specific time in the past:

z=0 today z=1 ~8Gyr ago z=6 ~13Gyr ago (age of the Universe <1Gyr!)



Observational facts

1- In all the cases where the inner core of a galaxy has been resolved (i.e. In nearby galaxies), a massive black hole (MBH) has been found in the center.

2- MBHs are believed to be the central engines of Quasars: the only viable model to explain this cosmological objects is by means of gas accretion onto a MBH.

3- Quasars have been discovered at z~7, their inferred masses are ~10⁹ solar masses!

THERE WERE 10⁹ SOLAR MASS BHs WHEN THE UNIVERSE WAS <1Gyr OLD!!!

Our aim is to understand the MBH formation and evolution and to assess the consequences for GW astronomy









Hubble Space Telescope Wide Field / Planetary Camera

Ground-Based Optical/Radio Image

HST Image of a Gas and Dust Disk



380 Arc Seconds 88,000 LIGHT-YEARS 17 Arc Seconds 400 LIGHT-YEARS

Quasar 3C175 YLA 6cm image (c) NRAO 1996





Core of Galaxy NGC 426I

Hubble Space Telescope Wide Field / Planetary Camera

Ground-Based Optical/Radio Image

HST Image of a Gas and Dust Disk



380 Arc Seconds 88,000 LIGHT-YEARS

1.7 Arc Seconds 400 LIGHT-YEARS





Quasar 3C175 YLA 6cm image (c) NRAO 1996



Cosmological structure formation

(Binney & Tremaine, 1987, chapter 9)

The Universe after the Big Bang was not completely uniform

The matter content was (and is) dominated by dark matter. The ratio dark matter/baryonic matter is ~10:1

Gravitational instabilities due to non uniform matter distribution cause the matter to condense until small regions become gravitationally bound

These regions then decouple themselves from the global expansion of the universe and collapse, forming what we call the *first galactic minihalos*.

The baryonic matter feels the gravitational potential of these halos and falls at their center, forming the first protogalaxies

This halos continuously form during the cosmic history and merge with each other in what we call the hierarchical scenario for galaxy formation.

Halo formation: spherical collapse

Consider a flat, matter dominated Universe, and consider a region which is slightly denser than the mean density.

The self-gravitational force of the sphere depends only on the matter inside the sphere itself (Birkhoff's theorem), and the overdensity behaves like a small closed Universe.



Schematic evolution:

- Density contrast grows as universe expands
- Perturbation "turns around" at R = R_{turn}, t = t_{turn}
- If exactly spherical, collapses to a point at t = 2 t_{turn}
- Realistically, bounces and virializes at radius R = R_{virial}

The typical halo mass is an increasing function of time: bottom-up or

HIERARCHICAL structure formation!

The halo mass function evolves in time (redshift) with larger halos forming at lower redshifts (later times).





(From de Lucia et al. 2006)

What happens to the baryons? In the early Universe most of the baryonic matter is in form of hot atomic (H) or molecular (H₂) Hydrogen.



Baryons need to cool down (i.e. loose energy) in order to condense in dense structures and form stars.

The only way to cool down is through transition between different atomic or molecular levels.

We need to excite high energy levels to radiate this energy away.

The only way is collisional excitation: we need high temperatures!!!

Atomic Hydrogen can cool only at temperatures>10⁴K, while H₂ can cool already at 10³K.

NOTE: Temperature increases with halo mass!

$$T_{\rm vir} = 1.98 \times 10^4 \ \left(\frac{\mu}{0.6}\right) \left(\frac{M}{10^8 \ h^{-1} \ M_{\odot}}\right)^{2/3} \left[\frac{\Omega}{\Omega(z)} \ \frac{\Delta_c}{18\pi^2}\right]^{1/3} \left(\frac{1+z}{10}\right) \ {\rm K}$$

The halo virial temperature is a function of the halo mass. At high z, we need $M>10^6$ solar masses to cool H₂, and $M>10^8$ solar masses to cool H.



Seed BH formation

Gas cools very slowly forming a stable disc First stars: maybe one star per galaxy, up to several hundred times larger than the sun

es

If the star is more massive than ~300 solar masses, it collapses into a black hole, ~200 times the mass of Sun



Globally unstable gas infalls rapidly toward the galaxy center and a supermassive star forms



The stellar core collapses into a small black hole, embedded in what is left of the star The black hole swallows the envelope growing up to ~ one million solar masses

Locally unstable gas flows toward the galaxy center Gas fragments into stars, and a dense star cluster forms



Stars merge into a very massive star that collapses into a black hole ~1000 times more massive than the Sun

Critically depends on: -content of H2 -vicinity of an ionizing source -fragmentation -metallicity



Seed BH mass function

Volonteri 2010



NOTE: The mass function can shift to lower values when wind mass loss and fragmentation are taken into account



HIERARCHICAL GALAXY EVOLUTION...

According to the Hierarchical scenario, protogalaxies formed in the first halos at high redshift merge feel each other gravitational attraction and merge together to form the galaxies we see today in the local Universe



From De Lucia et al. 2006

...+ M_{BH}- BULGE RELATIONS...

Massive black holes (MBHs) are ubiquitous in the galaxy centres. In the last decade, tight relation have been discovered, correlating the MBH mass with the host galaxy bulge mass (Magorrian et al. 1998) and with the host bulge velocity dispersion (Gebhardt et al. 2000, Ferrarese & Merrit 2000). This relation are a clear hint of a coevolution of MBHs and host galaxies.



...= HIERARCHICAL MBH EVOLUTION

GENERAL FRAMEWORK:

> The halo hierarchy can be traced backwards by means of EPS Monte-Carlo merger tree.

The semi-analytic code follows the accretion and the dynamical history of BHs in every single branch of the tree

The adopted threshold for density peaks hosting a seed ensures an occupation fraction of order unity today for halos more massive than 10¹¹M₀

(Volonteri, Haardt & Madau 2003)



In a ACDM cosmology

In a nutshell



(From de Lucia et al. 2006)



(Menou et al 2001, Volonteri et al. 2003)



(Ferrarese & Merritt 2000, Gebhardt et al. 2000)

In a nutshell





(Menou et al 2001, Volonteri et al. 2003)



(Ferrarese & Merritt 2000, Gebhardt et al. 2000)

*Where and when do the first MBH seeds form? *How do they grow along the cosmic history? *What is their role in galaxy evolution? *What is their merger rate? *How do they pair together and dynamically evolve?

Accretion

Gravitational instabilities drive cold gas toward the galactic nucleus, Gas forms a disk around the MBH, starting the accretion process.

Now consider a flux of proton & electrons with density ρ being accreted onto a BH of mass *M*. The accreting material emits radiation with a luminosity *L*. Equating the gravitational force (acting on the accreting material) to the force due to the radiation pressure (exerted by the outward radiation emitted by the accretion disk itself)

$$F_{\rm grav} = \frac{GM(m_e + m_p)}{r^2} \approx \frac{GMm_p}{r^2} \qquad F_{\rm rad} = p_{\rm rad} \ \sigma_T = \frac{L/c}{4\pi r^2} \sigma_T$$

one found an equilibrium condition (in the spherical limit), which is commonly known as *Eddington accretion limit*, described by the *Eddington luminosity*:

$$L = \epsilon \dot{M} c^2 \qquad L_{\rm Edd} = \frac{4\pi G M m_{\rm p} c}{\sigma_{\rm T}}$$



Dediction

L_{EDD}=1.38x10³⁸ erg/s for a solar mass BH and scales as the BH mass. A 10⁹ solar mass MBH shines with a luminosity of about 10⁴⁷ erg/s (10¹⁴ Suns or 1000 MWs)!!!!!

This imply an accretion in mass given by:

MBHs CAN EFFICIENTLY INCREASE THEIR MASS!!!!!!

$$\frac{dM}{dt} = 2.5 \times 10^{-8} \left(\frac{M}{\mathrm{M}_{\odot}}\right) \,\mathrm{M}_{\odot} \mathrm{yr}^{-1}$$

The natural timescale related to accretion is the Eddington timescale:

$$t_{\rm Edd} = \frac{Mc^2}{L_{\rm Edd}} = \frac{\sigma_T c}{4\pi G m_p} = 0.45 \text{Gyr} \qquad \dot{M} = (1-\epsilon) \dot{M}_{\rm acc} = \frac{(1-\epsilon)}{\epsilon} f_{\rm Edd} \frac{M}{t_{\rm Edd}}$$

This defines the basic equation of mass growth via accretion

$$M(t) = M_0 e^{\frac{(1-\epsilon)}{\epsilon} f_{\text{Edd}} \frac{(t-t_0)}{t_{\text{Edd}}}}$$

Although often set to 0.1, ϵ is in fact an important parameter that depends on the spin. What is it?

$$\Delta E = -\frac{GmM}{2r_{\rm rim}} \Longrightarrow L = -\frac{dE}{dt} = \frac{G\dot{M}M}{2r_{\rm rim}}, \qquad L = \frac{1}{4\beta}\dot{M}c^2 = \epsilon\dot{M}c^2$$

 β =3 for a Sch. BH, β =1 for a max spinning BH and prograde accretion. The GR calculation gives

$$a = 0 \to \epsilon \approx 0.06 \to M = M_0 \ e^{\frac{t}{3 \times 10^7 \text{yr}}},$$
$$a = 0.998 \to \epsilon \approx 0.42 \to M = M_0 \ e^{\frac{t}{3 \times 10^8 \text{yr}}}$$

No problem accreting MBHs to 10⁹ solar masses by z=0, but what about z>7 QSOs? **Evidence that MBHs grow mostly via radiative efficient accretion comes from the Soltan argument (1982).**

By measuring the luminosity function of quasars, one can compute the energy density due to the light emitted by accreting MBHs

$$e = \frac{4\pi}{c} \int_0^\infty (1+z) dz \int_0^\infty n(S,z) S \ dS$$

An energy density corresponds to an accreted mass density via

$$\rho_{\rm acc} = \frac{1-\epsilon}{\epsilon} \frac{e}{c^2}$$



The luminosity function of quasars can be measured empirically so that the estimate of the accreted mass density can be compared to the current mass density in MBHs (which can be also measured):

$$\rho_{\rm acc} \approx 2.2 \times 10^5 \left(\frac{0.1}{\epsilon}\right) \frac{M_{\odot}}{Mpc^3} \qquad \rho_{\rm BH} \approx 3 - 5 \times 10^5 \frac{M_{\odot}}{Mpc^3}$$

About half quasars are obscured! Which brings the two estimates to match quite well.

Mergers













I-Dynamical friction: 10kpc-1pc

Consider a BH with mass $M_{\rm BH}$ moving with velocity V in a surrounding distribution of field star with a density ρ_* and a Maxwellian velocity distribution with dispersion σ . The drag exerted by the stars on the BH is given by:

$$\mathbf{F}_{\rm DF} = -4\pi \ln \Lambda G^2 M_{\rm BH}^2 \rho_* \left[\operatorname{erf} \left(\frac{V}{\sqrt{2}\sigma} \right) - \left(\sqrt{\frac{2}{\pi}} \frac{V}{\sigma} \right) \exp \left(-\frac{V^2}{2\sigma^2} \right) \right] \frac{\mathbf{V}}{V^3}$$

- in the limit V->0 this force is proportional to V
- in the limito of $V >> \sigma$ this force is proportional to $1/V^2$
- the drag is maximum for $V=\sigma$

In a gaseous medium the formula is similar:

$$\begin{split} \mathbf{F}_{\mathrm{DF}}^{\mathrm{gas}} &= -4\pi \ln \left[\frac{b_{\max}}{b_{\min}} \frac{(\mathcal{M}^2 - 1)^{1/2}}{\mathcal{M}} \right] G^2 \, M_{\mathrm{BH}}^2 \rho_{\mathrm{gas}} \frac{\mathbf{V}}{V^3}, \quad \text{for} \quad \mathcal{M} > 1 \\ \mathbf{F}_{\mathrm{DF}}^{\mathrm{gas}} &= -(4/3)\pi G^2 \, M_{\mathrm{BH}}^2 \rho_{\mathrm{gas}} \mathcal{M}^3 \mathbf{V} / V^3 \propto M_{\mathrm{BH}}^2 \rho_{\mathrm{gas}} \, \mathbf{V} / c_{\mathrm{s}}^3 \text{ for } \mathcal{M} \ll 1 \end{split}$$

but now $\mathcal{M} = V/c_s$ is the gas speed of sound. Again the drag is maximum when $V=c_s$, and is comparable to the stellar case.





Colpi & Dotti 2009








2a-stellar scattering



A star on a intersecting orbit receive a kick taking away from the binary an amount of energy of the order $(3/2)Gm_*\mu_{\rm BH}/a$

This energy, and the relative angular momentum carried away, can be used to define dimensionless rate that describe the evolution of the binary.

$$\frac{dN}{dt} = n\Sigma\sigma = \frac{2\pi G(M_1 + M_2 + m_3)na}{\sigma}$$

2a-stellar scattering



A star on a i an amount o

This energy used to defi binary.



WFPC2 captures a SMBH binary kicking stars out of the bulge

FIG. 7.— Cartoon showing a pair of supermassive black holes kicking stars away as they dance towards coalescence at the centre of a galaxy. Credit: Paolo Bonfini.

orbit with tricity **e** and velocity <u>V</u>

nary

be

$$\frac{da}{dt} = \frac{da}{dt}\Big|_{3b} + \frac{da}{dt}\Big|_{gw} = -Aa^2 - \frac{B}{a^3},$$

$$Triaxialitik eeps the hardening the evolution of the$$

а

Triaxiality of the merger remnant keeps the 'loss cone full' and the hardening rate ~constant

The evolution of the binary can be simply obtained by combining stellar and GW hardening (e.g. AS & Khan 2015)



MBHB dynamics (BBR 1980)



RADIUS, R [parsec]

2b-Circumbinary disk-driven binaries

Secondary black hole

Primary black hole

Circumbinary disk (CBD) Gas inflows with a constant accretion rate. Its change in angular momentum is

$$\frac{dL}{dt} = -\dot{m}\sqrt{GMr_{\rm gap}}$$

The binary acts as a dam holding the gas at r_{gap} . Therefore is injecting in the disk an angular momentum equal and opposite to the above

©K.H Accretion disks Therefore the angular momentum dLof the binary also evolve as dL

$$\frac{dL}{dt} = -\dot{m}\sqrt{GMr_{\rm gap}}$$

Using $L = \mu \sqrt{GMa}$ and assuming that the mass ratio does not change one get the equation

$$\frac{da}{a} = -2\sqrt{2}\frac{dM}{\mu}$$

The binary makes ~3 e-folds by accreting a mass equal to mu. Assuming Eddington limited accretion this happens in ~4 x 10⁷ yrs. (Dotti+15)

Supermassive black hole triplets



Figure 1. Cartoon representation of how triple MBH interactions are treated in the semianalytic model described in Section 2.



Figure 2. Same as figure 1, but for quadruple interactions.



merger timescales comparable with galaxy subsequent merger timescale

There is a concrete possibility of injecting a third MBH in the system when the binary is still 'slowly hardening' (Hoffmann & Loeb 2007; Amaro-Seoane, AS, et al. 2010; Bonetti+16; Bonetti+17a,b; Ryu+17)

Integration of the 3-body dynamics



We designed a code for evolving MBHB triplets including

-PN dynamics up to 2.5 order, including all terms consistently derived from the 3-body Hamiltonian

-Dynamical friction (Chandrasekhar 1943)

-Stellar hardening (Sesana 2006)

-Spherical external potential

The code has been extensively tested reproducing results from the literature.



It can handle complex chaotic dynamics



But do we see them?





1 kpc: double peaked NL (Comerford 2013)



10 pc: double radio cores (Rodriguez 2006)



1 pc: -shifted BL (Tsalmatzsa 2011) -accelerating BL (Eracleous 2012)



0.01 pc: periodicity (Graham 2015)



0.0pc:-X-shaped sources (Capetti 2001) -displaced AGNs (Civano 2009)

MBHB dynamics (BBR 1980)



Gravitational waves

If the binary overcome the final parsec problem then it coalesces on a timescale given by:

$$t_{\rm GW} = \frac{5c^5}{256G^3} \frac{a^4}{M_1 M_2 MF(e)} \approx 0.25 \text{Gyr} \left(\frac{MM_1 M_2}{10^{18.3} \text{ M}_{\odot}^3}\right)^{-1} F(e)^{-1} \left(\frac{a}{0.001 \text{ pc}}\right)^4$$

producing the loudest gravitational wave signals in the Universe!

Simulated LISA data stream at merger event, two 10⁵M_☉ BH at z=5 including simulated noise (S/N~500)





amplitude characteristic



MBHB population models

Semianalytic models for galaxy and MBH formation and evolution (Barausse).

The explored scenarios cover a wide range of merger histories:

- -Heavy seeds no time delays
- -Heavy seeds time delays
- -PopIII seeds time delays



What LISA will measure

Assuming 4 years of operation:

- ~100+ detections
- ~100+ systems with sky localization to 10 deg2





~100+ systems with individual masses determined to 1%

- ~50 systems with primary spin determined to 0.01
- ~50 systems with secondary spin determined to 0.1
- ~50 systems with spin direction determined within 10deg
- ~30 events with final spin determined to 0.1

MBH astrophysics with GW observations

(b × b)

P(P1 × P)

Astrophysical unknowns in MBH formation scenarios

- 1- MBH seeding mechanism (heavy vs light seeds)
- 2- Metallicity feedback (metal free vs all metalliticies)
- 3- Accretion efficiency (Eddington?)
- 4- Accretion geometry (coherent vs. chaotic)

looktekik time (dvr)

CRUCIAL QUESTION: Given a set of LISA observation of coalescing MBH binaries, what astrophysical information about the underlying population can we recover?

Create catalogues of observed binaries including errors from eLISA observations and compare observations with theoretical models



AS et al. 2011, see also Plowman et al 2011

Resolving ringdown modes: BH spectroscopy

(Berti et al. 2016)



LIGO will not enable BH spectroscopy on individual BHB mergers

Voyager/ET type detectors are needed

eLISA will enable precise BH spectroscopy on few to 100 events/yr also at very high redshifts









Associated electromagnetic signatures?

In the standard circumbinary disk scenario, the binary carves a cavity: no EM signal (Phinney & Milosavljevic 2005). However, all simulations (hydro, MHD) showed significant mass inflow (Cuadra et al. 2009, Shi et al 2011, Farris et al 2014...)



Simulations in hot gaseous clouds. Significan flare associated to merger (Bode et al. 2010, 2012, Farris et al 2012)

t=0M







Simulations in disk-like geometry. Variability, but much weaker and unclear signatures (Bode et al. 2012, Gold et al. 2014)

Full GR force free electrodynamics (Palenzuela et al. 2010, 2012)





Cosmology with MBHBs



(Tamanini+ 2017)

Pulsar timing arrays

(Perrodin & AS 2017)



Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

Millisecond pulsars



What is pulsar timing

Pulsars are neutron seen through their regular radio pulses

Pulsar timing is the art of measuring the time of arrival (ToA) of each pulse and then subtracting off the expected time of arrival given by a theoretical model for the system

1-Observe a pulsar and measure the ToAs

2-Find the model which best fits the ToAs

3-Compute the timing residual R

R=ToA-ToA_m

If the timing solution is perfect (and observations noiseless), then R=0. *R* contains all uncertainties related to the signal propagation and detection, plus the effect of unmodelled physics, like (possibly) gravitational waves





Pulsar timing model



Which can be inverted to get the time of arrival of the N-th pulse

$$t_{\rm arr} = \nu^{-1} N - \Delta_{\rm R}(u) - \Delta_{\rm E}(u) + \Delta_{\rm S}(u) - \frac{1}{2} \dot{\nu} \nu^{-3} N^2 - \frac{1}{6} \ddot{\nu} \nu^{-4} N^3 + \dots$$

tSSB=t_{arr}+t_{clock}+ t_{Earth}+DM/*f*²

(Hobbs+ 2006)

The timing model includes: (Will 2006) -Roemer, Einstein and Shapiro delays. -pulsar frequency and freq derivatives -pulsar position and proper motion -dispersion measure -clock corrections and Earth position wrt SSB



(Kramer & Lorimer 2005)

Dispersion measure due to scattering of radio photons by the interstellar medium



Shapiro delay due to photon passage in the gravitational potential well of the companion



 $\Delta_{\rm S} = -2r\ln[1 - e\cos E - s(\sin\omega(\cos E - e) + \sqrt{1 - e^2}\cos\omega\sin E)]$



In the end you are left with your residual

R=ToA-ToA_m

If everything is properly taken into account, *R* has the properties of a white noise and is described by its rms



Equation of dopplershift for spacecraft ranging (Estabrook 1975)

$$z(t) = \frac{\nu(t) - \nu_0}{\nu_0} = \frac{\delta\nu(t)}{\nu_0}$$

$$z(t) = \frac{1}{2} p_i p_j \int_{t_p}^t dt' \frac{\partial}{\partial t'} h_{ij} [t', (t-t_p)\hat{p}]$$

For the system on the right this becomes

$$z(t) = \frac{1}{2}(1 - \cos\theta)[h_{xx}(t_p) - h_{xx}(t)]$$

Where
$$t - t_p \equiv \tau = (L/c)(1 + \hat{\Omega} \cdot \hat{p})$$

What PTA measures is the integral of this dephasing, which we call the 'residual'

$$r(t) = \int_0^t dt' z(t', \hat{\Omega}).$$





This can be generalized as we did for interferometers

$$\hat{\Omega} = -(\sin\theta\cos\phi)\,\hat{x} - (\sin\theta\sin\phi)\,\hat{y} - \cos\theta\hat{z}$$
$$\hat{p} = (\sin\theta_p\cos\phi_p)\,\hat{x} + (\sin\theta_p\sin\phi_p)\,\hat{y} + \cos\theta_p\hat{z},$$
$$h_{ij}(t,\hat{\Omega}) = e_{ij}^+(\hat{\Omega})h_+(t,\hat{\Omega}) + e_{ij}^\times(\hat{\Omega})h_\times(t,\hat{\Omega})$$
$$e_{ij}^+(\hat{\Omega}) = \hat{m}_i\hat{m}_j - \hat{n}_i\hat{n}_j,$$
$$e_{ij}^\times(\hat{\Omega}) = \hat{m}_i\hat{n}_j + \hat{n}_i\hat{m}_j.$$



$$\hat{m} = (\sin\phi\cos\psi - \sin\psi\cos\phi\cos\theta)\hat{x} - (\cos\phi\cos\psi + \sin\psi\sin\phi\cos\theta)\hat{y} + (\sin\psi\sin\theta)\hat{z},$$
$$\hat{n} = (-\sin\phi\sin\psi - \cos\psi\cos\phi\cos\theta)\hat{x} + (\cos\phi\sin\psi - \cos\psi\sin\phi\cos\theta)\hat{y} + (\cos\psi\sin\theta)\hat{z}.$$

$$z(t,\hat{\Omega}) = \frac{1}{2} \frac{\hat{p}^i \hat{p}^j}{1+\hat{p}^i \hat{\Omega}_i} \left\{ e_{ij}^+(\hat{\Omega}) \left[h_+(t_p,\hat{\Omega}) - h_+(t,\hat{\Omega}) \right] - e_{ij}^\times(\hat{\Omega}) \left[h_\times(t_p,\hat{\Omega}) - h_\times(t,\hat{\Omega}) \right] \right\}$$

which can be written in compact form as

$$z(t,\hat{\Omega}) = \sum_{A} F^{A}(\hat{\Omega}) [h_{A}(t_{p},\hat{\Omega}) - h_{A}(t,\hat{\Omega})]$$

where

$$F^{+}(\hat{\Omega}) = \frac{1}{2} \frac{(\hat{m} \cdot \hat{p})^{2} - (\hat{n} \cdot \hat{p})^{2}}{1 + \hat{\Omega} \cdot \hat{p}}$$
$$F^{\times}(\hat{\Omega}) = \frac{(\hat{m} \cdot \hat{p})(\hat{n} \cdot \hat{p})}{1 + \hat{\Omega} \cdot \hat{p}}.$$



Effect of gravitational waves

The GW passage causes a modulation of the observed pulse frequency

$$\frac{\nu(t) - \nu_0}{\nu_0} = \Delta h_{ab}(t) \equiv h_{ab}(t_{\rm p}, \hat{\Omega}) - h_{ab}(t_{\rm ssb}, \hat{\Omega})$$

$$R(t) = \int_0^T \frac{\nu(t) - \nu_0}{\nu_0} dt$$



(Sazhin 1979, Hellings & Downs 1983, Jenet et al. 2005, AS et al. 2008, 2009)

R~h/(2πf)

$$= \frac{\mathcal{M}^{5/3}}{D} [\pi f(t)]^{-1/3}$$

$$\simeq 25.7 \left(\frac{\mathcal{M}}{10^9 M_{\odot}}\right)^{5/3} \left(\frac{D}{100 \,\mathrm{Mpc}}\right)^{-1}$$

$$\times \left(\frac{f}{5 \times 10^{-8} \,\mathrm{Hz}}\right)^{-1/3} \,\mathrm{ns}$$



characteristic amplitude

Single MBHB timing residuals





The expected GW signal in the PTA band



The GW characteristic amplitude coming from a population of circular MBH binaries

$$h_c^2(f) = \int_0^\infty dz \int_0^\infty d\mathcal{M} \, \frac{d^3 N}{dz d\mathcal{M} d \ln f_r} h^2(f_r)$$
$$\delta t_{\rm bkg}(f) \approx h_c(f) / (2\pi f)$$

Theoretical spectrum: simple power law

(Phinney 2001)

$$h_c(f) = A\left(\frac{f}{\mathrm{yr}^{-1}}\right)^{-2/3}$$



The signal is contributed by extremely massive $(>10^8M_{\odot})$ relatively low redshift (z<1) MBH binaries (AS et al. 2008, 2012)






We are looking for a correlated signal



For a superposition of waves we can write

$$z(t) = \sum_{A=+,\times} \int_{-\infty}^{\infty} df \int d^2 \hat{\Omega} F^A(\hat{\Omega}) \tilde{h}_A(f,\hat{\Omega}) e^{-2\pi i f t} \left[1 - e^{-2\pi i f \tau}\right]$$

Unpolarized, isotropic, stationary signal

$$\langle \tilde{h}_A^*(f,\hat{\Omega})\tilde{h}_A'(fi,\hat{\Omega}')\rangle = \delta(f-f')\frac{\delta^2(\hat{\Omega},\hat{\Omega}')}{4\pi}\delta_{AA'}\frac{1}{2}S_h(f) \\ \langle z_a(t)z_b(t)\rangle = \frac{1}{2}\int_{-\infty}^{\infty} df S_h(f)\int d^2\hat{\Omega}\frac{1}{4\pi}\sum_{A=+,\times}F_a^A(\hat{\Omega})F_b^A(\hat{\Omega}).$$

The integral over dOmega gives

$$C(\zeta_{ab}) = \frac{1}{4} \left[1 + \frac{\cos \zeta_{ab}}{3} + 4(1 - \cos \zeta_{ab}) \ln \left(\sin \frac{\zeta_{ab}}{2} \right) \right]$$

We can then get the correlation in the residual by integrating to find

We finally get

1.1

1.11

$$r_{ab} = \Gamma(\zeta_{ab}) \int_0^\infty df P_h(f)$$

$$\Gamma(\zeta_{ab}) = \frac{3}{2}C(\zeta_{ab})(1+\delta_{ab})$$

Is known as Helling & Downs curve

We are looking for a correlated signal



Correlation

(Hellings & Downs 1983)













Likelihood function

All search methods are based on the likelihood function, describing the probability that the residuals contain a signal of some sort described by certain parameters

$$\mathcal{L}(\vec{\delta t}|\vec{\theta},\vec{\lambda}) = \frac{1}{\sqrt{(2\pi)^{n-m}det(G^T C G)}} \times \exp\left(-\frac{1}{2}(\vec{\delta t}-\vec{r})^T G(G^T C G)^{-1} G^T(\vec{\delta t}-\vec{r})\right)$$

The signal is contained in the correlation matrix C which is a function of the signal power as a function of sky location and the 'antenna beam patterns'

$$\Gamma_{ab} = \frac{3}{8\pi} (1 + \delta_{ab}) \int_{S^2} d\hat{\Omega} \ P(\hat{\Omega}) \sum_{\mathcal{A}} F_a^{\mathcal{A}}(\hat{\Omega}) F_b^{\mathcal{A}}(\hat{\Omega})$$

For an isotropic background this takes the form below, known as the 'Hellings & Downs ' curve:

$$\Gamma(\theta_{mn}) = \frac{3}{8} \left[1 + \frac{\cos \theta_{mn}}{3} + 4(1 - \cos \theta_{mn}) \ln \left(\sin \frac{\theta_{mn}}{2} \right) \right] (1 + \delta_{mn})$$



Emergence of the GW signal in theory



A worldwide observational effort

EPTA/LEAP (Large European Array for Pulsars)



NANOGrav (North American nHz Observatory for Gravitational Waves)

PPTA (Parkes Pulsar Timing Array)



A worldwide observational effort



A worldwide observational effort



The overall GW signal

Population parameters

1-Galaxy merger rate <----> MBHB merger rate affects the number of sources at each frequency ---> No

2-MBH mass - merging galaxy relation affects the mass of the sources ----> M



$$\begin{split} h_{c}^{2}(f) &= \int_{0}^{\infty} dz \int_{0}^{\infty} dM_{1} \int_{0}^{1} dq \frac{d^{4}N}{dz dM_{1} dq dt_{r}} \frac{dt_{r}}{d\ln f_{\mathrm{K},r}} \times \\ h^{2}(f_{\mathrm{K},r}) \sum_{n=1}^{\infty} \frac{g[n, e(f_{\mathrm{K},r})]}{(n/2)^{2}} \delta\left[f - \frac{nf_{\mathrm{K},r}}{1+z}\right]. \end{split}$$

$$h_{c}(f) \propto (n_{0}^{1/2} f^{-\gamma} M_{c}^{5/6})$$

Local dynamics

1-Accretion (when? how?)

affects the mass of the sources $--> M_c$

2-MBHB - environment coupling (gas & stars)

affects the chirping rate of the binaries ---> γ affects the eccentricity ---> chirping rate ----> γ & single source detection



Uncertainty in the GW background shape



STELLAR DRIVEN BINARIES assuming stars are supplied to the binary loss cone at a constant rate:



$$dt/d\ln f \propto f^{2/3} M_1^{2/3}$$

 $h_c \propto M_1^2 q f.$

GAS DRIVEN BINARIES self-consistent solution for the binary-disk interaction with no leakage in the cavity:

$$\frac{da}{dt} = \frac{2\dot{M}}{\mu}(aa_0)^{1/2}.$$

$$dt/d\ln f \propto f^{-1/3} M_1^{1/6}$$

$$h_c \propto M_1^{7/4} q^{3/2} f^{1/2}$$

Transition frequency

$$f_{\text{star/GW}} \approx 5 \times 10^{-9} M_8^{-7/10} q^{-3/10}$$

 $f_{\text{gas/GW}} \approx 5 \times 10^{-9} M_8^{-37/49} q^{-69/98}$



(Kocsis & AS 2011, AS 2013, Ravi et al. 2014, McWilliams et al. 2014)

Theory and observations progression



Example of non-detection (EPTA, Lentati et al. 2015)



Current limits not quite constraining

-Comprehensive set of semianalytic models anchored to observations of galaxy mass function and pair fractions (AS 2013, 2016) -Include different BH mass-galaxy relations

-Include binary dynamics (coupling with the environment/eccentricity)



(Middleton et al., 2018)

The nature of the signal



*It is not Gaussian *Single sources might pop-up ***The distribution of** the brightest sources might well be anisotropic

Continuous GW analysis

$$\mathcal{L}(\vec{\delta t}|\vec{\theta},\vec{\lambda}) = \frac{1}{\sqrt{(2\pi)^{n-m}det(G^T C G)}} \times \exp\left(-\frac{1}{2}(\vec{\delta t}-\vec{r})^T G(G^T C G)^{-1} G^T(\vec{\delta t}-\vec{r})\right)$$

The correlation matrix C is now defined by a deterministic signal that for circular GW driven binaries takes the form:

$$r_a(t) = r_a^p(t) - r_a^e(t),$$

where

$$r_a^e(t) = \frac{\mathcal{A}}{\omega} \left\{ (1 + \cos^2 \iota) F_a^+ \left[\sin(\omega t + \Phi_0) - \sin \Phi_0 \right] + 2 \cos \iota F_a^{\times} \left[\cos(\omega t + \Phi_0) - \cos \Phi_0 \right] \right\},$$
$$r_a^p(t) = \frac{\mathcal{A}_a}{\omega_a} \left\{ (1 + \cos^2 \iota) F_a^+ \left[\sin(\omega_a t + \Phi_a + \Phi_0) - \sin(\Phi_a + \Phi_0) \right] + 2 \cos \iota F_a^{\times} \left[\cos(\omega_a t + \Phi_a + \Phi_0) - \cos(\Phi_a + \Phi_0) \right] \right\},$$

The signal depends on several parameters, and consists of two superimposed sinusoids

$$\mathcal{A} = 2 \frac{\mathcal{M}_c^{5/3}}{D_L} (\pi f)^{2/3}$$



Limits on continuous GWs

(EPTA, Babak et al. 2015)

| Search ID | Noise treatment | N pulsars | N parameters | Signal model | Likelihood |
|---------------------|--------------------|-----------|--------------|--------------|--|
| Fp_ML | Fixed ML | 41 | 1 | E+P NoEv | Maximized over 4 constant amplitudes plus pulsar phase |
| Fp | Sampling posterior | 41 | 1 | E+P NoEv | Maximized over 4 constant amplitudes plus pulsar phase |
| Fe | Fixed ML | 41 | 3 | Е | Maximized over 4 constant amplitudes |
| Bayes_E | Fixed ML | 41 | 7 | Е | Full |
| Bayes_EP | Fixed ML | 6 | 7+2	imes 6 | E+P Ev | Full |
| Bayes_EP_NoEv | Fixed ML | 41 | 7 | E+P NoEv | Pulsar phase marginalization |
| Bayes_EP_NoEv_noise | Searched over | 6 | 7+5	imes 6 | E+P NoEv | Pulsar phase marginalization |



Astrophysical implications

The array sensitivity is function of the sky location, we can build sensitivity skymaps -14.00Sky sensitivity at f = 7 nHz 60° -14.0811012 + 530745° -14.1630° Coma 15° Virgo. -14.2411713+1774 12 h 20 h 6 h 0° 1744-1134 10613-8200 -14.32-15° 11600-3053 -30° -14.4011909 3744 -45° -14.48-60° -75° -14.56



Data are not yet very constraining, we can rule out very massive systems to ~200Mpc, well beyond Coma

Constraining astrophysical candidates

-Graham et al. 2015: 111 candidates from CRTS -Charisi et al 2016: 33 candidates from PTF -All candidates are individually consistent with PTA limits -The implied total signal is in tension with PTA limits at 2 – 3 sigma level (Sesana et al. 2018)







Limits published after 2015 are not solid:

- 1- Shannon et al 2015 → essentially a single pulsar limit. This is a problem since you have to model the pulsar red noise and if your array is dominated by a single pulsar you can never know whether its red noise is intrinsic. → 'over fitting'
- 2- Arzoumanian 2016, 2018 → Issues with solar system Ephemeridis. The data show some evidence of correlated red signal, but it can be absorbed in uncertanties in the SSE
- **3-** Since quite some time a common red signal has been present in several PTA data but it's nature hasn't be assessed (see point 2)

NOTE:

The choice of the prior in your analysis matters. When you think you don't have a signal in the data, you use a log uniform prior in the amplitude to place an upper limit, which has the effect to likely push your UL down.

So it is possible that by assuming there is no signal in the data, the recent UL have been overestimated

NANOGrav 12.5 year analysis



Full Bayesian analysis of 43 pulsars. Schemes to account for SSE and other noises

Clear detection of a common red process.

If this was a GWB, then A~2x10⁻¹⁵

$$h_c(f) = A\left(\frac{f}{\mathrm{yr}^{-1}}\right)^{-2/3}$$



Monopolar and Dipolar correlations seem disfavored.

However no evidence of HD correlation.

Interesting, seen in several PTA datasets, needs further investigation

MORE TO COME! Stay tuned!!

Signal interpretation arXiv:2011.01246

We now suppose for the sake of the argument that the signal is of GW origin.

A number of interpretations have been put forward in the literature including:

- first-order phase transitions
- cosmic strings
- domain walls
- large amplitude curvature perturbations
- primordial black holes
- inflation

The signal is indeed fully consistent with an astrophysical population of MBHBs. This interpretation must therefore be preferred by virtue of Occam razor (Middleton+20)





234k GWB realizations from Rosado et al 2020 Frequency binning of NANOGrav Powerlaw fit to the 5 lowest frequency bins Use the parametric model of Chen+19 to describe the spectrum as a function of astrophysical observables:

$$\begin{split} h_c^2(f) = &\int dz \int d\mathcal{M} \frac{d^2 n}{dz d\mathcal{M}} h_{c,\text{fit}}^2 \left(f \frac{f_{p,0}}{f_{p,t}} \right) \\ & \times \left(\frac{f_{p,t}}{f_{p,0}} \right)^{-4/3} \left(\frac{\mathcal{M}}{\mathcal{M}_0} \right)^{5/3} \left(\frac{1+z}{1+z_0} \right)^{-1/3} \end{split}$$





Can place unique astrophysical constraints on the merging MBHB population:

-MBHB do merge in nature! -typical merger timescale is <3Gyr -High MBH-bulge mass relations are favoured

The future



MeerKAT, South Africa (2017)

The future



FAST, China (2017)

The future



Square Kilometre Array (SKA, 2021+)
The future



Habemus GWs!

We see black hole binaries (BHB) and neutron star (NS) binaries coalescing for the first time (several Abbott+ 2016 2017)

- -GRB-NS merger connection -Heavy element production -NS EoS -First tests of GR in the strong field regime -Interesting astrophysical information
- (masses, spins)
- \rightarrow Formation scenario?











